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JULY 1975

A STUDY OF THE G-H-K
TRACKING FILTER

BY

R. J. POLGE
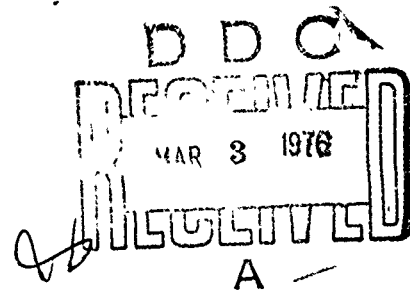B. K. BHAGAVAN

FINAL TECHNICAL REPORT
VOLUME I

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PREFACE

The work performed by the Communications group of the University of Alabama in Huntsville for the U. S. Army Missile Command (MICOM) under contract DAAH01-71-C-1181, during the period June 1, 1974 to July 31, 1975 consists of two tasks. The first task is documented in UAH Research Report No. 176 entitled, "A Study of the G-H-K Tracking Filter" and the second task will be documented in a UAH Research Report entitled, "Quantization Effects in Digital MTI Filters". This effort will be denoted as fourth-year effort because it is an extension of a contract on EAR simulation studies started on May 18, 1971. The goals of these four consecutive efforts are reviewed briefly, to place them in the proper perspective.

The overall objective of the first year effort was to develop, implement and exercise a detailed digital computer simulation of the experimental Array Radar (EAR) system in order to support and complement the experimental work being performed by MICOM personnel. The specific objectives of the study were to: (1) duplicate in a software form the operation of system hardware and its performance as measured in the test bed, (2) to simulate proposed changes of the hardware and predict performance improvement, and (3) to experiment with alternate digital and data processing techniques.

The objectives of the second year effort were to: (1) refine and update the baseline EAR simulation program and develop subprograms to study critical components of the system, (2) expand baseline EAR simulation programs to include multiple target handling, (3) investigate via simulation a constant-false-alarm-rate (CFAR) processor, and (4) investigate the feasibility of glint error reduction through the use of frequency agility, pulse compression or both.

The objectives of the third-year effort were (1) A/D converter evaluation by digital computer analysis and (2) an expansion of the scope of EAR simulation studies to include (a) design of experiments and (b) an investigation of command guidance requirements.

In the fourth-year effort presently ending, the goal of the first task was to help the Army in the selection of an adaptive tracking filter which could track a maneuvering target. The goals of the second task were to investigate quantization effects for various types and realizations of digital MTI filter and to recommend a filter which represents the best compromise between performance and hardware requirements.

Huntsville, Alabama
July 1975

Robert J. Polge

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A B S T R A C T

This report is concerned with the investigation of the g-h-k filter for tracking maneuvering targets. The selection of the filter coefficients is based on the amounts of noise and maneuver, and other system considerations such as critical damping or best transient response for specified smoothing. Several filter initialization schemes were tested. For low acceleration maneuvers, a considerable amount of smoothing can be achieved without losing track. However, in order to track severely maneuvering targets, one must select coefficients which give a faster transient response at the expense of smoothing capability. Therefore, it is logical to use an adaptive filter with a good smoothing capability when the target is not maneuvering and a fast response during a target maneuver. Clearly, the main problem is to detect the maneuver in a reasonable amount of time. This can be done using a simplified matched filter followed by a threshold detector. The proposed adaptive filter was evaluated through computer simulation using typical trajectories. The performance of the adaptive filter is limited by the number of samples required by the detection filter and could probably be improved using a more complex maneuver detection filter.

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CHAPTER 1
INTRODUCTION

1.1 <u>Prologue</u>

In radar tracking systems, the raw measurements of a target position must be processed to provide the predicted position of the target at the next time a radar action is scheduled for this target. This estimate, in radar coordinates, is then used to steer the antenna beam in the predicted direction at the next tracking instant. In order to maintain the track, the prediction accuracy must be high enough that the target will be located within the beam and within the range bin centered around the predicted range. Prediction requires not only a knowledge of the present position, which can be measured, but also an estimate of the rate (and possibly higher derivatives) of the target motion, which may not be measured by the radar. In view of the fact that the measurements are obtained at discrete intervals of time, some form of digital filter suggests itself for the prediction system.

1.2 <u>Tracking Filters</u>

In order to design a tracking filter, it is necessary to have an adequate mathematical model for the motion of the target. In practice, the trajectory will not be a polynomial for all time, but can be piecewise approximated over short intervals by a polynomial of suitable degree. Tracking filters [1] may be non-recursive (open-loop) or recursive (closed-loop). A well-known filter of the non-recursive type is the polynomial filter [2]. However, the most widely used tracking filters are of the recursive variety. Prominent among these are the Kalman filter [3,4], the g-h filter [4-6] and the g-h-k filter [7,8], all of which are based on polynomial models. The Kalman filter minimizes the mean-squared error if the model is exact and the order of the filter is one more than the

degree of the polynomial. The feedback gains of a Kalman filter are different at each step. The g-h filter is a second-order filter and is a simple observer [9] for constant velocity targets, whereas the g-h-k filter is a third-order observer for constant acceleration targets. While the gains of a Kalman filter are derived on the basis of statistical considerations, those of the g-h and g-h-k filters are chosen via practical considerations. In their most common forms, the g-h and g-h-k filters have constant feedback gains.

The design of a tracking filter must take into account two sources of error, modeling and noise. The modeling or the dynamic error is caused by the target motion not exactly fitting the assumed model. This is especially severe when the target is maneuvering, either due to atmospheric turbulence or due to an evasive maneuver. The noise is due to the radar measurement errors, and can be adequately modeled by a white noise sequence. The requirements for rapid dynamic response and smoothing of the noise are always in conflict and a suitable compromise is necessary in designing the filter. The response to a changing input improves as the feedback gains are made larger, but this results in a decreasing ability to smooth out random fluctuations. Other factors to be considered in designing a tracking filter include the choice of the coordinate system and the tracking interval. Among the various coordinate systems available [10] are (i) the spherical coordinates, range, azimuth and elevation, (ii) the rectangular coordinates and (iii) a mixed coordinate system. The tracking interval may be constant or varying.

1.3 Scope and Organization of the Report

The main objective of the task reported here is to study the applicability of g-h-k filters to track maneuvering and non-maneuvering

targets. The study consists of two parts: (i) the selection of the coefficients g, h and k based on analytical considerations and (ii) the development of a procedure to dynamically adjust the coefficients to adapt to changes in t'.e trajectory.

Throughout this report, it is assumed that the tracking interval remains constant, that the prediction is performed in the range, azimuth and elevation coordinates and that the three filters are independent of each other. It will also be assumed that only position measurements and no velocity measurements are available.

The g-h-k tracking filter is briefly reviewed in Chapter 2. Input-output relations for the filter, in both the time-domain and the frequency-domain are presented.

The selection of the g-h-k filter coefficients is discussed in Chapter 3. Equations are derived to express two of the coefficients, h and k, in terms of the third coefficient g assuming one of the following criteria: (i) critical damping, (ii) best transient response for specified smoothing (optimal), and (iii) use of steady-state Kalman gains. The choice of g should reflect the compromise between speed of response and smoothing capability.

The performance of fixed coefficient g-h-k filters is studied in Chapter 4 via simulation. The filters are applied to maneuvering and non-maneuvering trajectories. The peak and RMS errors are computed for different maneuvers, different noise strengths and for various values of the filter parameter g. Both optimal filters and critically damped filters are considered.

A scheme for the detection of a target maneuver is proposed in Chapter 5. It is based on detecting a bias in the filter residual sequence and is an approximation to the decision theoretical method. Also presented

in Chapter 5 is a procedure to adjust the filter coefficients dynamically when a maneuver is detected. The method is simulated and validated using typical trajectories.

Chapter 6 contains the summary and conclusions of the report along with recommendations for future work.

CHAPTER 2
THE G-H-K FILTER

2.1   Introduction

The g-h-k filter, also known as the α-β-γ filter, is a poly-nomial predictor-corrector filter of the third order and is of the linear recursive type. It is simply an observer designed to reconstruct the position, velocity and acceleration of a constant acceleration target based on position measurements. Therefore, it can track a constant acceleration trajectory with no systematic errors in the steady state. In addition to the predicted estimate, the g-h-k filter also provides a smoothed estimate of the present position which may be useful for guidance and weapon control functions. In its standard form, the g-h-k filter uses only the position measurements, and must be modified to include any available Doppler measurements.

2.2   The Filter Equations [10]

Let x be, in general, the coordinate in which the filter operates, so that x represents either range, azimuth or elevation. Assume that the smoothed estimates of the position, velocity and acceleration at time t_n are known and denoted as $\hat{x}(n/n)$, $\hat{\dot{x}}(n/n)$ and $\hat{\ddot{x}}(n/n)$. Then, the predicted state at time t_{n+1} is computed as,

$$\hat{x}(n+1/n) = \hat{x}(n/n) + T_n \hat{\dot{x}}(n/n) + \frac{T_n^2}{2} \hat{\ddot{x}}(n/n) \tag{2.1}$$

$$\hat{\dot{x}}(n+1/n) = \hat{\dot{x}}(n/n) + T_n \hat{\ddot{x}}(n/n) \tag{2.2}$$

$$\hat{\ddot{x}}(n+1/n) = \hat{\ddot{x}}(n/n) \tag{2.3}$$

where

$$T_n = t_{n+1} - t_n \quad . \tag{2.4}$$

After the position measurement $x_m(n+1)$ has been received at time t_{n+1}, the predicted state can be corrected resulting in the smoothed estimate at t_{n+1}.

$$\hat{x}(n+1/n+1) = \hat{x}(n+1/n) + g_{n+1} \, (x_m(n+1) - \hat{x}(n+1/n)) \tag{2.5}$$

$$\hat{\dot{x}}(n+1/n+1) = \hat{\dot{x}}(n+1/n) + \frac{h_{n+1}}{T_{n+1}} \, (x_m(n+1) - \hat{x}(n+1/n)) \tag{2.6}$$

$$\hat{\ddot{x}}(n+1/n+1) = \hat{\ddot{x}}(n+1/n) + \frac{2k_{n+1}}{T^2_{n+1}} \, (x_m(n+1) - \hat{x}(n+1/n)) \tag{2.7}$$

where g_{n+1}, h_{n+1} and k_{n+1} are the gain coefficients. Denoting the vector made up of x, $\dot{x}$ and $\ddot{x}$ by $\underline{X}$, $i.e.$,

$$\underline{X} = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} \quad , \tag{2.8}$$

equations (2.1) to (2.7) can be written as

$$\hat{\underline{X}}(n+1/n) = \Phi(n) \, \hat{\underline{X}}(n/n) \tag{2.9}$$

$$\hat{\underline{X}}(n+1/n+1) = \hat{\underline{X}}(n+1/n) + K(n+1)[x_m(n+1) - H \, \hat{\underline{X}}(n+1/n)] \tag{2.10}$$

where

$$H = [\ 1 \quad 0 \quad 0 \ ] \tag{2.11}$$

$$K = \begin{bmatrix} g_{n+1} \\ \dfrac{h_{n+1}}{T_{n+1}} \\ \dfrac{2k_{n+1}}{T^2_{n+1}} \end{bmatrix} \tag{2.12}$$

and the state transition matrix $\Phi(n)$ is given by

$$\Phi(n) = \begin{bmatrix} 1 & T_n & \dfrac{T^2_n}{2} \\ 0 & 1 & T_n \\ 0 & 0 & 1 \end{bmatrix} \quad . \tag{2.13}$$

Since only the predicted estimates are of interest here, (2.9) and (2.10) can be combined as

$$\hat{\underline{X}}(n+1/n) = \Phi(n)[I-K(n)H] \, \hat{\underline{X}}(n/n-1) + \Phi(n)K(n)x_m(n) \tag{2.14}$$

$$= \Psi(n) \, \hat{\underline{X}}(n/n-1) + \Phi(n)K(n)x_m(n) \tag{2.15}$$

where

$$\Psi(n) = \Phi(n)[I-K(n)H] \tag{2.16}$$

and   I is the third order identity matrix.

In the analysis in this report, it will be assumed that the tracking interval T_n, and the gain coefficients g_n, h_n and k_n are constant. This will eliminate the dependency of Ψ, Φ and K on the time variable n, and (2.15) becomes,

$$\hat{\underline{X}}(n+1/n) = \Psi \ \hat{\underline{X}}(n/n-1) + \Gamma \ x_m(n) \tag{2.17}$$

where

$$\Psi = \begin{bmatrix} 1 - g - h - k & T & T^2/2 \\ -(h+2k)/T & 1 & T \\ -2k/T^2 & 0 & 1 \end{bmatrix} \tag{2.18}$$

$$\Gamma = \begin{bmatrix} g + h + k \\ (h + 2k)/T \\ 2k/T^2 \end{bmatrix} \quad . \tag{2.19}$$

2.3   Frequency Domain Representation

The system represented by (2.17) is linear and shift invariant with $x_m(n)$ as the input sequence, $\hat{\underline{X}}(n+1/n)$ as the vector output sequence. Therefore, it can be analyzed in the frequency domain using its z-transfer function. To compute the transfer function, a z-transform is taken on both sides of (2.17) which yields,

$$Z[\hat{\underline{X}}] = z^{-1} \Psi Z[\hat{\underline{X}}] + \Gamma Z[x_m] \tag{2.20}$$

where the operator $Z[\cdot]$ denotes the z-transform. Equation (2.20) can be written as,

$$Z[\hat{\underline{X}}] = (z \ I-\Psi)^{-1} \ z\Gamma Z[x_m] \quad . \tag{2.21}$$

Using (2.18) and (2.19), the above equation can be simplified to give the transfer functions for the predicted position, velocity and acceleration.

Each transfer function is of the type,

$$H(z) = \frac{a_3 z^3 + a_2 z^2 + a_1 z}{z^3 + b_2 z^2 + b_1 z + b_0} \qquad (2.22)$$

where the coefficients are given in Table 2.1.

TABLE 2.1   G-H-K FILTER TRANSFER FUNCTION COEFFICIENTS

COEFFICIENTS \ OUTPUT TYPE	a_3	a_2	a_1	b_2	b_1	b_0
PREDICTED POSITION	g+h+k	k-2g-h	g			
PREDICTED VELOCITY	$\frac{h+2k}{T}$	$\frac{-2h-2k}{T}$	$\frac{h}{T}$	g+h+k-3	3-2g-h+k	g-1
PREDICTED ACCELERATION	$\frac{2k}{T^2}$	$\frac{-4k}{T^2}$	$\frac{2k}{T^2}$			

It can be seen that the characteristic polynomial (denominator of the transfer function) is the same for all outputs and is given by

$$c(z) = z^3 - (3-g-h-k)z^2 + (3-2g-h+k)z - (1-g) . \qquad (2.23)$$

Using standard z-domain techniques [11], it can be shown that the system is stable if the following conditions are satisfied:

$$g > 0$$

$$h > 0$$

$$k > 0 \qquad (2.24)$$

$$2g+h \le 4$$

$$2g > k$$

$$g(h+k) > 2k .$$

It can also be shown that the poles of the transfer function move towards the unit circle as the coefficients g, h and k are decreased. This results in a reduced bandwidth which provides good smoothing at the same time producing a sluggish dynamic response. On the other hand, an increase in the filter coefficients improves the dynamic response at the cost of poor noise smoothing.

CHAPTER 3
SELECTION OF THE FILTER COEFFICIENTS

3.1   Introduction

It can be seen from the filter equations presented in Chapter 2, that the g-h-k filter is completely defined by the three coefficients g, h and k. As was discussed earlier, in the selection of the coefficients, one must compromise between the conflicting requirements of good noise smoothing (narrow bandwidth) and of good transient capability (wide bandwidth). A very practical selection scheme would be to express two of the coefficients in terms of the other based on theoretical considerations and maintain only one parameter for the compromise. The choice of a value for this parameter must depend on the system application where the relative importance of smoothing and dynamic response can be ascertained. In this chapter, expressions for h and k, in terms of g, are derived based on three different criteria. The merits and demerits of each are briefly discussed. A fourth coefficient selection scheme is described which should be useful during heavy transients.

3.2   Critically Damped G-H-K Filters

Since the g-h-k filter is a third order observer designed to track a constant acceleration trajectory, the steady-state tracking error for such a motion is zero in the absence of noise. However, tracking errors do occur during transients caused by improper initialization and by changes in the trajectory. The amount of time required for the transient errors to die down, is a function of the damping of the filter. An under-damped filter undergoes periodic oscillations before settling down. Under-damping is caused by the presence of a pair of complex conjugate poles and can be avoided by forcing all the poles to be real. Three real and unequal poles produce no oscillations but the response will be slow, resulting in

overdamping. Critical damping provides a fast response while still preventing oscillations.

Critical damping of a third order system can be achieved by forcing all the poles to be positive, real and equal. This can be accomplished by selecting the coefficients g, h and k such that the characteristic equation given by (2.23) has three real and equal roots. Assuming the roots are given by $z = \beta$, critical damping is obtained if the following equation is satisfied:

$$(z-\beta)^3 = z^3 - (3-g-h-k)z^2 + (3-2g-h+k)z - (1-g). \tag{3.1}$$

Equating the coefficients of like powers of z on the two sides of (3.1) yields,

$$3\beta = 3-g-h-k \tag{3.2}$$

$$3\beta^2 = 3-2g-h+k \tag{3.3}$$

$$\text{and} \qquad \beta^3 = 1-g. \tag{3.4}$$

Solving (3.2)-(3.4) for g, h and k yields,

$$g = 1 - \beta^3 \tag{3.5}$$

$$h = 1.5(1 - \beta^2)(1-\beta) \tag{3.6}$$

$$k = 0.5 \; (1-\beta)^3 \qquad . \tag{3.7}$$

Given the value of g, (3.5) can be solved for β which is then used to determine the values of h and k from (3.6) and (3.7). Alternatively, β can be retained as the compromising parameter. Stability requires that

$$0 \le \beta \le 1 \; . \tag{3.8}$$

When $\beta = 0$, no smoothing is applied to the data and the amount of smoothing increases as β increases. When $\beta = 1$, all the filter coefficients are zero and the filter output becomes independent of the input. Figure 3.1 shows the variation of h and k as g varies from 0 to 1.
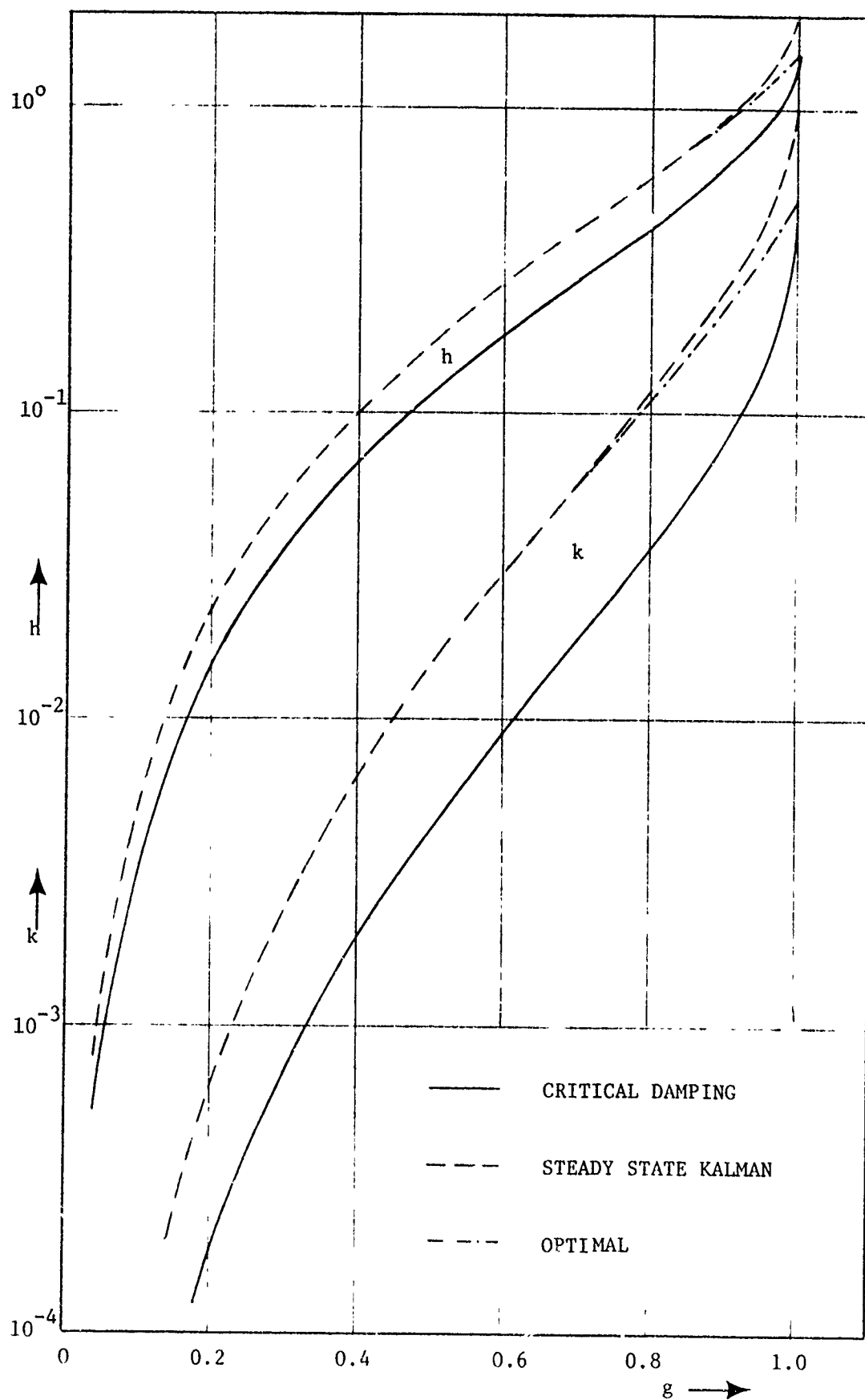
FIGURE 3.1   VARIATION OF h AND k WITH RESPECT TO g

The stringent condition that there be no oscillations could result in an excessive transient period, especially following a severe maneuver. In such cases, it may be advantageous to allow some oscillations in order to improve the dynamic response.

3.3 The Optimal Filter

Here again, only one parameter will be left free for the compromise. But for every value of this parameter, the other two will be chosen to give the maximum noise smoothing for a given transient capability. It is in this sense that the filter is optimal and the synthesis procedure is similar to the one used in [6] for g-h filters. Before attempting a compromise, it is necessary to define suitable measures of noise reduction and transient performance.

For noise smoothing, the performance measure will be the variance reduction ratio, *i.e.*, the steady-state variance in position output when the input is a unit variance white noise sequence. If $\{g(k)\}$ is the impulse response sequence for the predicted position, then the output for an input sequence of $\{f(k)\}$ is

$$c(k) = \sum_{j=0}^{k} g(j) f(k-j) \quad . \quad (3.9)$$

The autocorrelation of the output is,

$$K(n) = E[c(k)c(k+n)]$$

$$= \sum_{j=0}^{k} \sum_{i=0}^{k+n} g(i)g(j)E[f(k-j)f(k+n-i)] \quad (3.10)$$

where $E[\cdot]$ denotes the statistical expectation operator. If the input is a zero mean independent sequence, (3.10) becomes, in steady state,

$$K(n) = \sum_{j=0}^{\infty} g(j)g(j+n) \quad . \quad (3.11)$$

Thus, the variance of the output sequence is given by,

$$K(0) = \sum_{j=0}^{\infty} g^2(j) \ . \tag{3.12}$$

For transient performance, the measure is based on the capability to follow a constant acceleration input. The input considered is,

$$x(k) = (k^2 + k)/2 \ . \tag{3.13}$$

If the predicted position output sequence is $\{r(k)\}$, the transient performance measure is,

$$D = \sum_{k=0}^{\infty} [x(k) - r(k-1)]^2 \ . \tag{3.14}$$

This simple input is chosen to facilitate analysis, however, it retains the important feature of constant acceleration.

The optimum filter is synthesized by selecting the impulse response sequence $\{g(k)\}$ so as to minimize $K(0)$ for a given D, or vice versa. The problem can be solved in the framework of constrained optimization in which one wishes to minimize

$$J = K(0) + \lambda D \tag{3.15}$$

where λ is the Lagrange multiplier.

The z-transform of the input sequence (3.13) is,

$$X(z) = \frac{z^2}{(z-1)^3} \ . \tag{3.16}$$

Therefore, if $G(z)$ is the transfer function (z-transform of the sequence $\{g(k)\}$), the z-transform of the output sequence is given by

$$R(z) = X(z)\, G(z)$$
$$\frac{z^2}{(z-1)^3}\, G(z) \ . \tag{3.17}$$

Rearranging (3.17) yields,

$$G(z) = \frac{z^3 - 3z^2 + 3z - 1}{z^2}\, R(z) \tag{3.18}$$

An inverse z-transform of (3.18) gives,

$$g(k) = r(k+1) - 3r(k) + 3r(k-1) - r(k-2) \quad . \tag{3.19}$$

Using (3.19), (3.12) and (3.14), (3.15) becomes,

$$J = \sum_{k=0}^{\infty} [\{r(k+1) - 3r(k) + 3r(k-1) - r(k-2)\}^2 \tag{3.20}$$

$$+ \lambda\{x(k) - r(k)\}^2] \; .$$

In order to find the first variation of J, the optimal r(k) is perturbed by

$$r(k) \rightarrow r(k) + \varepsilon \, h(k)$$

where ε is a constant and h(k) is an arbitrary variation. The optimal r(k) is found by equating the first variation of J to zero, *i.e.*,

$$\left. \frac{\partial J[r(n) + \varepsilon \, h(n)]}{\partial \varepsilon} \right|_{\varepsilon = 0} = 0 \quad . \tag{3.21}$$

Substituting (3.20) in (3.21) yields,

$$\sum_{k=0}^{\infty} [\{r(k+1) - 3r(k) + 3r(k-1) - r(k-2)\}\{h(k+1) - 3h(k) + 3h(k-1) - h(k-2)\}$$
$$+ \lambda\{x(k) - r(k-1)\}\{-h(k)\}] = 0 \; . \tag{3.22}$$

The realizability of the filter requires that,

$$\left. \begin{array}{l} h(k) = 0 \\ \text{and} \qquad r(k) = 0 \end{array} \right\} \quad \text{for } k \leq 0 \quad . \tag{3.23}$$

Using (3.23) and rearranging the summation indices, (3.22) becomes,

$$\sum_{k=0}^{\infty} h(k)[r(k+2) - 6r(k+1) + 15r(k) - 20r(k-1) + 15r(k-2)$$
$$- 6r(k-3) + r(k-4) - \lambda\{x(k) - r(k-1)\}] = 0 \quad . \tag{3.24}$$

Since the variation h(k) is arbitrary, the quantity inside the brackets must be identically zero, *i.e.*,

$$r(k+2) - 6r(k+1) + 15r(k) - 20r(k-1) + 15r(k-2) - 6r(k-3)$$

$$+ r(k-4) + \lambda r(k-1) - \lambda x(k) = 0 \tag{3.25}$$

$$\text{for all } k > 0.$$

Because the left hand side of (3.25) is zero for all k, its z-transform can have poles only on or outside the unit circle. Thus,

$$F(z) = (z^2 - 6z + 15 - 20z^{-1} + 15z^{-2} - 6z^{-3} + 1 + \lambda z^{-1})R(z) - \lambda x(z) \qquad (3.26)$$

has all its poles on or outside the unit circle. Rewriting (3.26), yields

$$F(z) = \frac{(z-1)^6 + \lambda z^3}{z^4} R(z) - \lambda X(z). \qquad (3.27)$$

Since X(z) given by (3.16) has no poles inside the unit circle, the realizable poles of R(z) must match the roots of

$$(z-1)^6 + \lambda z^3 = 0 . \qquad (3.28)$$

It can be easily seen that if $z = a$ is a root of (3.28), then $z = 1/a$ is also a root. Let a, b and c be the three realizable poles of G(z), *i.e.*, a, b and c are the roots of the characteristic equation

$$z^3 - (3 - g - h - k)z^2 + (3 - 2g - h + k)z - (1 - g) = 0 . \qquad (3.29)$$

Since the realizable poles of R(z) and G(z) are the same, it follows that,

$$(z-a)(z-b)(z-c)(z-1/a)(z-1/b)(z-1/c) = (z-1)^6 - \lambda z^3 \qquad (3.30)$$

where

$$a + b + c = 3 - g - h - k \qquad (3.31)$$

$$a b c = 1 - g \qquad (3.32)$$

$$ab + bc + ac = 3 - 2g - h + k . \qquad (3.33)$$

Equating coefficients of like powers of z on both sides of (3.30) and using (3.31) - (3.33), yields

$$g^2 = h(2-g) - gk \qquad (3.34)$$

and $$h^2 = k^2 + 4 gk. \qquad (3.35)$$

The above equations can be solved for h and k in terms of g, giving

$$h = \frac{(2g^3 - 4g^2) + \sqrt{4g^6 - 64g^5 + 64g^4}}{8(1-g)} \qquad (3.36)$$

$$\text{and} \quad k = \frac{h(2-g) - g^2}{g} \quad . \tag{3.37}$$

This method of selecting the coefficients was based on minimizing K(0) for a given D. Equations (3.36) and (3.37) are the expressions for h and k in terms of g. The optimal g depends on λ, which is selected in such a way that the value of D equals the specified value. Figure 3.1 shows the variation of h and k as g varies from 0 to 1. Once again, high values of g result in poor smoothing and the amount of noise reduction increases as g decreases.

This set of coefficients can be shown to result in a slightly underdamped system. The optimality of the system depends on the validity of the trajectory model. Therefore, the filter is very well suited in situations where the motion is sufficiently steady. However, if the target acceleration is changing rapidly, its transient performance may not be adequate.

3.4 Use of Steady State Kalman Gains

In the case of the optimum filter described in the previous section, the squared error in the predicted position was minimized assuming a deterministic model. The Kalman filter [12] is based on a statistical model and minimizes the mean squared errors in the three outputs.

The message model under consideration is given by

$$\underline{X}(n+1) = \Phi \, \underline{X}(n) + \underline{W}(n) \tag{3.38}$$

where the vector $\underline{X}$ is made up of the position, velocity and acceleration as defined in (2.8) and $\{\underline{W}(n)\}$ is an independent vector noise sequence included to account for modeling inaccuracies. The state transition matrix Φ is

$$\Phi = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad . \tag{3.39}$$

The noise sequence $\{\underline{W}(n)\}$ is assumed to have zero mean and covariance matrix Q, *i.e.*,

$$E[\underline{W}(n)] \quad = \quad \underline{0}$$

$$E[\underline{W}(k)\underline{W}^T(n)] \; = \quad Q \qquad \text{if } n = k$$

$$\qquad\qquad\qquad = \; 0 \qquad \text{otherwise,} \tag{3.40}$$

where

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{12} & Q_{22} & Q_{23} \\ Q_{13} & Q_{23} & Q_{33} \end{bmatrix} .$$

The measurement sequence $x_m(n)$ is given by

$$x_m(n) = H \underline{X}(n) + v(n) \tag{3.41}$$

where

$$H = [1 \quad 0 \quad 0] \tag{3.42}$$

and $v(n)$ is a zero mean independent noise sequence with variance R. Since the Kalman filter algorithms have been widely published, only the final equations will be repeated here.

The algorithm for the predicted estimate at the (n+1) th stage based on measurements up to and including the n th stage is

$$\hat{\underline{X}}(n+1/n) = \Phi[I-K(n)H]\hat{\underline{X}}(n/n-1)+\Phi K(n)x_m(n) \quad . \tag{3.43}$$

The gain vector $K(n)$ is computed as

$$K(n) = P(n) \, H^T[H \, P(n)H^T + R]^{-1} \tag{3.44}$$

where $P(n)$ is the predicted error covariance,

$$P(n) = E[\{\underline{X}(n) - \hat{\underline{X}}(n/n-1)\}\{\underline{X}(n)-\hat{\underline{X}}(n/n-1)\}^T]. \tag{3.45}$$

The error covariance matrix is computed from the recursive relation,

$$P(n+1) = \Phi[I-K(n)H]P(n) \, \Phi^T + Q. \tag{3.46}$$

A casual examination of equations (2.14) and (3.46) reveals that the Kalman filter is identical to the g-h-k filter except for the way in which the gains are selected. The Kalman filter gain $K(n)$ is time varying as can be seen from (3.44) and (3.46). However, it can be shown that, after a large number of steps, the error covariance matrix and hence the

gain vector reach steady states after which their values remain constant. Therefore, this set of constant gains can be used as the feedback coefficients of the g-h-k filter.

Let P_{ij} and K_i denote respectively, the elements of the covariance matrix and the gain vector, $i.e.$,

$$P(n) = \begin{bmatrix} P_{11}(n) & P_{12}(n) & P_{13}(n) \\ P_{12}(n) & P_{22}(n) & P_{23}(n) \\ P_{13}(n) & P_{23}(n) & P_{33}(n) \end{bmatrix} \qquad (3.47)$$

and

$$K(n) = \begin{bmatrix} K_1(n) \\ K_2(n) \\ K_3(n) \end{bmatrix} . \qquad (3.48)$$

Equation (3.44) can now be expanded to give

$$K_1(n) = \frac{P_{11}(n)}{P_{11}(n) + R} \qquad (3.49)$$

$$K_2(n) = \frac{P_{12}(n)}{P_{11}(n) + R} \qquad (3.50)$$

$$K_3(n) = \frac{P_{13}(n)}{P_{11}(n) + R} . \qquad (3.51)$$

Using (3.49) - (3.51), (3.46) can be expanded to yield [8],

$$P_{11}(n+1) = K_1(n)R + 2 K_2(n) R T + K_3 R T^2 - P_{12}(n) K_2(n) T^2$$
$$- P_{13}(n) K_2(n) T^3 + P_{22}(n) T^2 + P_{23}(n) T^3$$
$$- 1/4 P_{13}(n) K_3(n) T^4 + 1/4 P_{33}(n) T^4 + Q_{11} \qquad (3.52)$$

$$P_{12}(n+1) + K_2(n) R + K_3(n) R T - P_{12}(n) K_2(n) T + P_{22}(n) T$$
$$+ 3/2 P_{23}(n) T^2 - 3/2 P_{12}(n) K_3(n) T^2$$
$$- 1/2 P_{13}(n) K_3(n) T^3 + 1/2 P_{33}(n) T^3 + Q_{12} \qquad (3.53)$$

$$P_{13}(n+1) = K_3(n) R - P_{12}(n) K_3(n) T + P_{23}(n) T + 1/2 P_{33}(n) T^2$$
$$- 1/2 P_{13}(n) K_3(n) T^2 + Q_{13} \qquad (3.54)$$

$$P_{22}(n+1) = -P_{12}(n) \, K_2(n) + P_{22}(n) + 2 \, P_{23}(n) \, T - 2P_{12}(n) \, K_3(n) \, T$$
$$-P_{13}(n) \, K_3(n) \, T^2 + P_{33}(n) \, T^2 + Q_{22} \qquad (3.55)$$

$$P_{23}(n+1) = -P_{12}(n) \, K_3(n) + P_{23}(n) - P_{13}(n) \, K_3(n) \, T + P_{33}(n)T + Q_{23} \qquad (3.56)$$

$$P_{33}(n+1) = -P_{13}(n) \, K_3(n) + P_{33}(n) + Q_{33} \quad . \qquad (3.57)$$

In order to find the steady state gains, it is necessary to assign values to the system covariance coefficients Q_{ij}. Assuming that the target is moving with constant acceleration except for an uncertainty in the acceleration, the covariance matrix Q can be adequately modelled as,

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_m^2 \end{bmatrix} \quad . \qquad (3.58)$$

The quantity σ_m^2 is the maneuver variance, and may be approximated from the maximum expected target jerk. Using Equation (3.58) and the fact that, in steady state,

$$P_{ij}(n+1) = P_{ij}(n) \quad \text{for i, j} = 1, \, 2, \, 3 \quad ,$$

Equations (3.49) - (3.57) can be manipulated to show that the steady state gains satisfy,

$$2 \, T \, K_3 = K_1(K_1 + K_2 \, T + K_3 \, \frac{T^2}{2}) \qquad (3.59)$$

$$K_2^2 = 2K_1 \, K_3 \qquad (3.60)$$

and

$$\frac{K_3^2}{1 - K_1} = \frac{\sigma_m^2}{R} \quad . \qquad (3.61)$$

The first two equations can be used to express two coefficients in terms of the third, which is then chosen using the last equation. Comparing (3.48) with the g-h-k filter coefficients, it can be seen that

$$K_1 = g$$
$$K_2 = h/T \qquad (3.62)$$
$$K_3 = \frac{2k}{T^2} \quad .$$

Use of (3.50) in (3.59) and (3.60) yields,

$$2h = g(g + h + k) \tag{3.63}$$

$$h^2 = 4 g k \tag{3.64}$$

which can be solved to give,

$$h = -2g + 4 - 4 \sqrt{1-g} \tag{3.65}$$

$$k = h^2/4g. \tag{3.66}$$

Since the main intent of this analysis is to express two coefficients in terms of the third, (3.61) will not be discussed further. Given a value of g, (3.53) and (3.54) can be used to compute the corresponding values of h and k. Figure 3.1 shows the variation of h and k as g varies from 0 to 1.

As can be seen, the steady state Kalman gains and the optimum gains are identical except at large values of g. Since the steady state gains are used, it should be expected that the filter has a poor transient performance. Further, the optimality of the Kalman filter depends entirely on the validity of the message model. Since the assumed model is not adequate when there is a severe maneuver, the filter will not perform satisfactorily. The coefficient selection is useful mainly during steady periods, where high noise smoothing, or a small value of g, is desired. However, since the gains are the same as the optimum gains for small values of g, the optimum gains could be used in all situations.

3.5   Coefficients to Improve Transient Performance

All the three selection schemes discussed so far were based on optimizing the steady-state performance. However, in cases where heavy transients are expected, it may be desirable to design the filter to provide a good transient response.

The method suggested here assumes the same message model, (3.38) - (3.42), as the Kalman filter. Using this model and the equation (2.17) of the g-h-k filter, it can be shown, by elementary linear system covariance analysis methods, that the error covariance matrix P follows the difference equation,

$$P(n+1) = \Psi P(n) \Psi^T + \Gamma R \Gamma^T + Q \quad . \tag{3.67}$$

The covariance matrix P is symmetric and hence has only six unknowns. Let $\underline{S}$ denote the vector made up of these six elements of P, $i.e.$,

$$\underline{S}(n) = \begin{bmatrix} P_{11}(n) \\ P_{12}(n) \\ P_{13}(n) \\ P_{22}(n) \\ P_{23}(n) \\ P_{33}(n) \end{bmatrix} \quad . \tag{3.68}$$

Using (2.18) and (2.19), after a modest amount of manipulations, (3.67) can be written as,

$$\underline{S}(n+1) = B \, C \, \underline{S}(n) + B \, \gamma + \sigma \tag{3.69}$$

where

$$B = \begin{bmatrix} 1 & 2T & T^2 & T^2 & T^3 & T^4/4 \\ 0 & 1 & T & T & 3T^2/2 & T^3/2 \\ 0 & 0 & 1 & 0 & T & T^2/2 \\ 0 & 0 & 0 & 1 & 2T & T^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.70}$$

$$C = \begin{bmatrix} (1-g)^2 & 0 & 0 & 0 & 0 & 0 \\ -h(1-g)/T & (1-g) & 0 & 0 & 0 & 0 \\ -2k(1-g)/T^2 & 0 & (1-g) & 0 & 0 & 0 \\ h^2/T^2 & -2h/T & 0 & 1 & 0 & 0 \\ 2kh/T^3 & -2k/T^2 & 0 & 0 & 1 & 0 \\ 4k^2/T^4 & 0 & -4k/T^2 & 0 & 0 & 1 \end{bmatrix} \tag{3.71}$$

$$\gamma = R \begin{bmatrix} g^2 \\ gh/T \\ 2gk/T^2 \\ h^2/T^2 \\ 2kh/T^3 \\ 4k^2/T^4 \end{bmatrix} \tag{3.72}$$

$$\text{and} \qquad \sigma = \begin{bmatrix} Q_{11} \\ Q_{12} \\ Q_{13} \\ Q_{22} \\ Q_{23} \\ Q_{33} \end{bmatrix} . \tag{3.73}$$

The method suggested here minimizes the cost function

$$J = \sum_{n=0}^{N} P_{11}(n) \tag{3.74}$$

$$= \sum_{n=0}^{N} A \underline{S}(n) \tag{3.75}$$

where
$$A = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] . \tag{3.76}$$

In (3.76), N is the number of steps over which the transient performance is optimized. A small N results in improved transient performance, whereas the filter reduces to the one described in Section 3.4 as N tends to infinity. Assuming the initial condition $\underline{S}(0)$ is given as $\underline{So}$, the minimization can be approached from an optimal control point of view. The Hamiltonian is given by

$$H = A \underline{S}(n) - \lambda^T(n+1) [B C \underline{S}(n) + B\gamma + \sigma] \tag{3.77}$$

where λ is the Lagrange multiplier vector. The optimal solution satisfies,

$$\underline{S}(n+1) = B C \underline{S}(n) + B\gamma + \sigma, \quad \underline{S}(0) = \underline{So} \tag{3.78}$$

$$\lambda(n) = A - C^T B^T \lambda(n+1), \quad \lambda(N+1) = 0 \tag{3.79}$$

$$\sum_{n=0}^{N} = \frac{\partial H}{\partial g} = 0 = \sum_{n=0}^{N} \lambda^T(n+1)[B \frac{\partial C}{\partial g} \underline{S}(n) + B\frac{\partial \gamma}{\partial g} + \sigma] \tag{3.80}$$

$$\sum_{n=0}^{N} \frac{\partial H}{\partial h} = 0 = \sum_{n=0}^{N} \lambda^T(n+1)[B\frac{\partial C}{\partial h} \underline{S}(n) + B\frac{\partial \gamma}{\partial h} + \sigma] \tag{3.81}$$

and $$\sum_{n=0}^{N} \frac{\partial H}{\partial k} = 0 = \sum_{n=0}^{N} \lambda^T(n+1)[B\frac{\partial C}{\partial k} \underline{S}(n) + B\frac{\partial \gamma}{\partial k} + \sigma] \ . \tag{3.82}$$

The optimization requires the solution of a two point boundary value problem. Analytical expressions for g, h and k cannot be obtained since the matrix C and the vector σ contain nonlinear functions of g, h and k. A simple steepest ascent algorithm similar to the one used in [13] can be utilized to solve the problem.

3.6   Conclusions

Three methods were presented where two of the filter coefficients are expressed in terms of the third in such a way that meaningful criteria are satisfied. The selection of the third coefficient is entirely dependent on the practical situation. All three schemes were derived using a constant acceleration trajectory. Assuming that the coefficients remained constant for all time, the methods attempted to optimize the performance for all time. However, in some cases where heavy transients are present, it may be desirable to optimize the performance over a short period of time. A coefficient selection scheme was presented where this is accomplished using optimal control principles.

CHAPTER 4
PERFORMANCE OF THE FIXED COEFFICIENT G-H-K FILTER

4.1   Introduction

As previously discussed, the choice of a value for g is based on a compromise between speed of response and noise smoothing capability. Another factor of importance is the selection of the initial state which is required to start the recursive filter. In order to evaluate the performance of the fixed coefficient g-h-k filter, it was simulated on a digital computer and applied to simulated measurements. The purpose of this experiment is twofold: (i) to select a filter initialization scheme which results in the best transient performance and (ii) to compare the performance of the filter, using several values of g and different coefficient selection schemes, for tracking trajectories with varying amounts of noise and maneuver. It is assumed throughout that the tracking interval is constant and equal to 0.1 sec. All the computer programs developed for this study are listed in Appendix A.

4.2   Measurement and Filter Simulations

4.2.1   Subroutine TRAJ

This program is used to generate samples of range and azimuth of a maneuvering or nonmaneuvering target. It is assumed that the target velocity, the tracking interval and the target altitude are constant. The simulated trajectory in the x-y plane consists of three segments. a straight line path from time $t = 0$ to $t = T_1$ seconds, (ii) a maneuver to the right from $t = T_1$ to $t = T_2$ at a constant acceleration and (iii) a straight line path from $t = T_2$ to $t = T_3$. Three typical trajectories which are used in the analysis are shown in Figure 4.1. Note that a nonmaneuvering trajectory can be obtained by setting $T_1 = T_2 = T_3$. In addition to the tracking interval DELT which is transmitted as an argument,
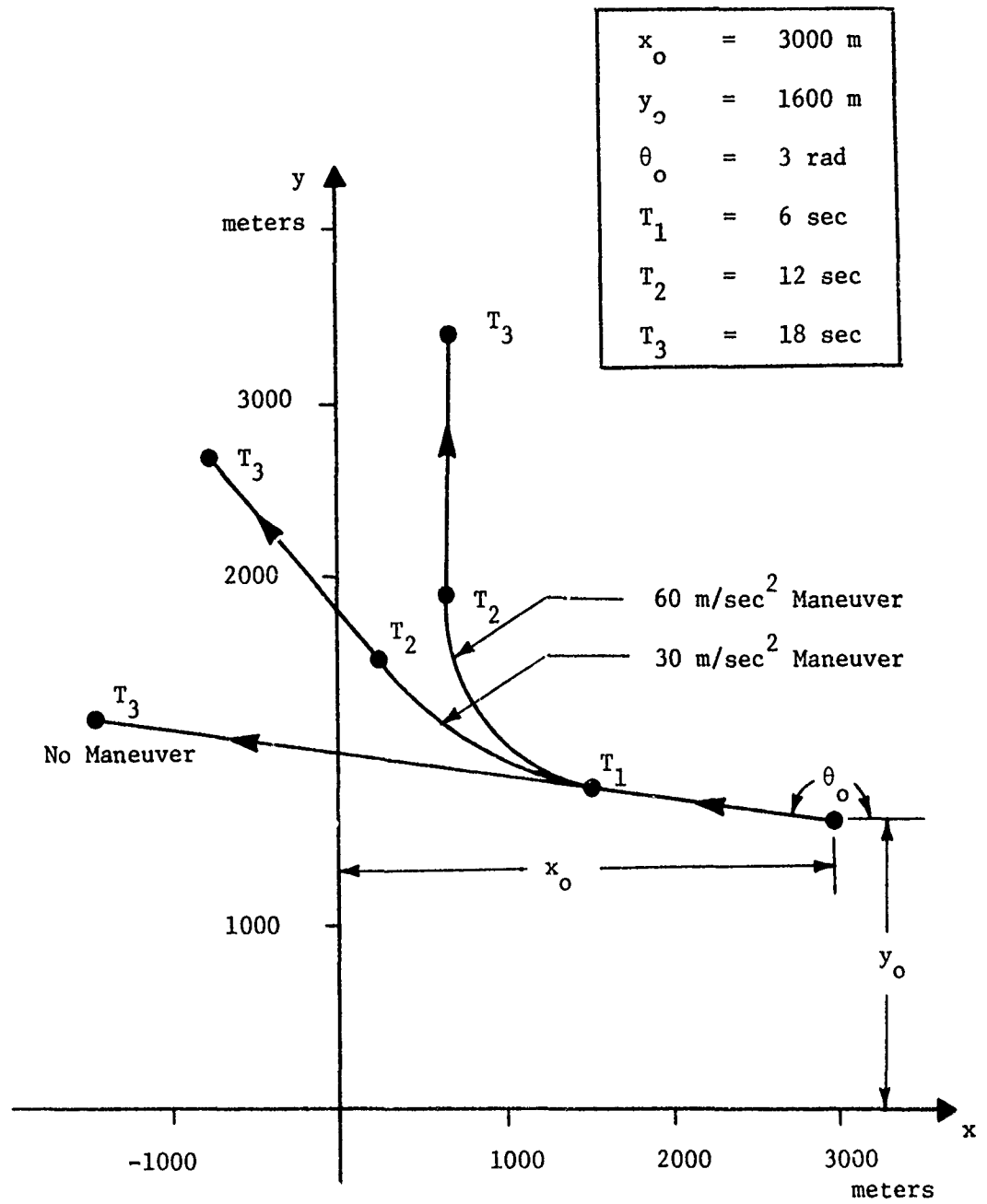
FIGURE 4.1   TRAJECTORIES USED IN THE ANALYSIS

the following inputs are read in:

XO, YO	the initial position of the target
Z	the constant altitude
VEL	the constant velocity
THETA	the initial angle measured from the x-axis
T1, T2, T3	the times corresponding to T_1, T_2 and T_3
AC	the constant radial acceleration during the maneuver.

The outputs of the program are the number of samples N, and two arrays, R and A, consisting of N samples each of the range and azimuth. All the outputs are returned through arguments.

The equations used are:

$$\left. \begin{array}{l} x(n) = XO + n.T.V \cos \theta \\ y(n) = YO + n.T.V \sin \theta \end{array} \right\} \quad \text{for } n\,T \leq T1 \tag{4.1}$$

$$r = V^2/a$$

$$\left. \begin{array}{l} \alpha(n) = V(n.t-T1)/r \\ x(n) = x(\frac{T1}{T})+r[\cos(\theta-\pi/2)-\sin(\alpha(n)+\pi-\theta)] \\ y(n) = y(\frac{T1}{T})+r[\sin(\theta-\pi/2)-\cos(\alpha(n)+\pi-\theta)] \end{array} \right\} \quad \text{for } T1<nT\leq T2 \tag{4.2}$$

$$\left. \begin{array}{l} x(n) = x(\frac{T2}{T})+(n.T-T2)V \cos(\theta-\alpha(\frac{T2}{T})) \\ y(n) = y(\frac{T2}{T})+(n.T-T2)V \sin(\theta-\alpha(\frac{T2}{T})) \end{array} \right\} \quad \text{for } T2<nT\leq T3 \tag{4.3}$$

where

$$T = DELT$$

$$\theta = THETA$$

$$V = VEL$$

$$a = AC$$

and   r = radius during maneuver.

The range and azimuth samples are calculated as,

$$R(n) = \sqrt{x^2(n) + y^2(n) + z^2} \qquad (4.4)$$

$$A(n) = \tan^{-1}(y(n)/x(n)). \qquad (4.5)$$

4.2.2 Subroutine NOISE

This program computes the simulated measurements by adding noise samples to the range and azimuth samples generated by TRAJ. The noise samples form a zero mean, Gaussian independent sequence with standard deviation RSTD and ASTD for range and azimuth respectively. In order to achieve a Monte-Carlo type simulation, this program is written so that it provides a different noise sequence every time. In addition to RSTD and ASTD, the inputs include the arrays R and AZ which contain samples of range and azimuth and N the number of samples. The outputs are the arrays XMR and XMA consisting of N samples of the simulated measurements in range and azimuth. All inputs and outputs are transmitted as arguments. The noise samples are generated using the subroutine RANDN of the UNIVAC 1108 library.

4.2.3 Subroutine GHKFIL

This program simulates the g-h-k filter in one coordinate based on the equations given in Chapter 2. An input INIT is used to select one of three different initialization schemes described in the next section. Other inputs are the array of measurements XM, the number of samples N, the tracking interval DELT, and the filter coefficients G, H and K. The outputs include the arrays XP, VP and AP containing predicted position, velocity and acceleration, respectively, and an array RES containing the residual sequence

$$RES(n) = x_m(n) - H \hat{\underline{X}}(n/n-1). \qquad (4.6)$$

All inputs and outputs are transmitted as arguments.

4.3 Filter Initialization

As can be seen from the equations in Chapter 2, the implementa-

tion of the recursive filter requires that the initial states be specified. It is important to choose these states properly since it is undesirable to have large transients. Assuming that three measurements have been made, three different initialization schemes are considered:

$$
(1) \quad
\begin{cases}
\hat{x}(3/3) = x_m(3) \\[2ex]
\hat{\dot{x}}(3/3) = \dfrac{x_m(3) - x_m(2)}{T} \\[2ex]
\hat{\ddot{x}}(3/3) = 0
\end{cases}
\qquad (4.7)
$$

$$
(2) \quad
\begin{cases}
\hat{x}(3/3) = x_m(3) \\[2ex]
\hat{\dot{x}}(3/3) = \dfrac{x_m(3) - x_m(2)}{T} \\[2ex]
\hat{\ddot{x}}(3/3) = \dfrac{x_m(3) - 2x_m(2) + x_m(1)}{T}
\end{cases}
\qquad (4.8)
$$

$$
(3) \quad
\begin{cases}
\hat{x}(3/3) = \dfrac{5x_m(3) + 2x_m(2) - x_m(1)}{T} \\[2ex]
\hat{\dot{x}}(3/3) = \dfrac{x_m(3) - x_m(1)}{2T} \\[2ex]
\hat{\ddot{x}}(3/3) = 0
\end{cases}
\qquad (4.9)
$$

The first set of equations is derived assuming a straight line between the second and third samples. In the second scheme, the position and velocity are based on a straight line between the second and third samples while the acceleration assumes piecewise straight lines between adjacent samples. The third alternative is derived through a least squares straight line fit between the three measurements. By setting INIT = 1,2, or 3, while calling subroutine GHKFIL, the corresponding initialization procedure can be utilized.

In order to compare the three initialization schemes, a main program TSTINI was written and exercised on a nonmaneuvering trajectory. Since the simulation results depend upon the particular noise sequence employed,

three different noise sequences were used. The basis of comparison was the absolute maximum value of the prediction error in the first fifteen tracking instances, both in range and angle. The results of this test are given in Table 4.1. The nonmaneuvering trajectory shown in Figure 4.1 was used along with the following parameters:

$$RSTD = 6m, \quad ASTD = 7.5 \text{ m.rad}$$

$$g = 0.5 \text{ with optimal selection of h and k.}$$

From the table it is apparent that initialization scheme 2 has the worst transient performance. This is probably because three samples are not sufficient to estimate the acceleration. Schemes 1 and 3 yield comparable results with scheme 3 being slightly better. Therefore, it will be used for initialization in the rest of the analysis.

4.4   G-H-K Filter Performance

In an attempt to evaluate the performance of the filter under a variety of conditions, a main program TSTGHK was written. The program was exercised for tracking three different trajectories, with two different noise levels. Several values of the coefficient g were tested and the coefficients h and k were selected using both the optimal and the critical damping methods. The bases of comparison are the absolute maximum error and the RMS error in both range and angle. Once again, the experiments were repeated for three different noise sequences. Typical results are given in Tables 4.2 through 4.7 where σ_R and σ_θ denote RSTD and ASTD respectively. Since noise sequence No. 3 provides a performance which is in between those of the other two sequences, all comparisons are made with respect to that sequence. For ease of comparison, all the results using noise sequence No. 3 are repeated in Table 4.8. The following different values of the parameters are used in the experiments whose results are shown in the tables.

TABLE 4.1 COMPARISON OF THREE TYPES OF INITIALIZATION

NOISE / INITIALIZATION		NOISE SEQUENCE # 1	NOISE SEQUENCE # 2	NOISE SEQUENCE # 3
SCHEME 1	PEAK RANGE ERROR meter	8.17	24.55	9.05
	PEAK ANGLE ERROR m.rad	17.14	17.08	14.11
SCHEME 2	PEAK RANGE ERROR meter	10.66	28.70	30.54
	PEAK ANGLE ERROR m.rad	102.26	40.05	29.56
SCHEME 3	PEAK RANGE ERROR meter	7.99	20.84	9.58
	PEAK ANGLE ERROR m.rad	9.13	14.89	11.98

TABLE 4.2   OPTIMAL FILTER, g = 0.3

MANEUVER	NOISE		$\sigma_R = 4$ m, $\sigma_\theta = 5$ m. rad			$\sigma_R = 8$ m, $\sigma_\theta = 10$ m. rad		
			SEQUENCE #1	SEQUENCE #2	SEQUENCE #3	SEQUENCE #1	SEQUENCE #2	SEQUENCE #3
NO MANEUVER	RANGE ERROR meter	PEAK	6.02	10.99	6.12	11.50	22.49	12.43
		RMS	2.45	2.76	2.57	4.77	5.63	4.99
	ANGLE ERROR m.rad	PEAK	8.83	11.81	9.38	17.90	23.5	19.4
		RMS	2.59	3.33	2.94	5.14	6.64	5.99
30 m/sec² MANEUVER	RANGE ERROR meter	PEAK	8.96	10.99	8.44	12.92	22.49	11.79
		RMS	3.54	3.68	3.11	5.65	6.31	5.21
	ANGLE ERROR m.rad	PEAK	9.16	12.16	9.38	18.23	23.85	19.4
		RMS	2.59	3.58	3.24	5.07	6.84	6.23
60 m/sec² MANEUVER	RANGE ERROR meter	PEAK	13.5	15.49	13.04	17.45	22.49	16.39
		RMS	4.68	4.9	4.15	6.56	7.28	5.88
	ANGLE ERROR m.rad	PEAK	8.75	12.49	9.58	17.82	24.18	19.4
		RMS	2.76	3.91	3.74	5.07	7.07	6.63

TABLE 4.3   CRITICALLY DAMPED FILTER, $g = 0.3$

MANEUVER	NOISE		$\sigma_R = 4$ m, $\sigma_\theta = 5$ m. rad			$\sigma_R = 8$ m, $\sigma_\theta = 10$ m. rad		
			SEQUENCE #1	SEQUENCE #2	SEQUENCE #3	SEQUENCE #1	SEQUENCE #2	SEQUENCE #3
NO MANEUVER	RANGE ERROR meter	PEAK	6.18	8.01	8.29	10.76	15.66	14.95
		RMS	2.69	2.48	3.07	4.47	4.91	5.08
	ANGLE ERROR m.rad	PEAK	8.51	10.24	5.78	16.04	19.87	11.73
		RMS	2.76	3.31	2.41	5.05	6.34	4.86
MANEUVER 30 m/sec²	RANGE ERROR meter	PEAK	12.78	12.08	11.97	17.81	16.73	16.06
		RMS	5.85	5.35	5.42	7.22	6.88	6.63
	ANGLE ERROR m.rad	PEAK	9.14	10.61	8.64	17.14	20.23	12.71
		RMS	3.16	4.01	3.09	5.25	6.91	5.34
MANEUVER 60 m/sec²	RANGE ERROR meter	PEAK	18.84	17.98	19.09	21.51	22.47	22.02
		RMS	8.67	8.18	7.91	9.93	9.41	8.66
	ANGLE ERROR m.rad	PEAK	10.66	13.14	10.02	18.66	20.57	15.11
		RMS	3.96	5.07	4.39	5.65	7.74	6.38

TABLE 4.4   OPTIMAL FILTER, g = 0.5

MANEUVER	NOISE		σR = 4 m, σθ = 5 m. rad			σR = 8 m, σθ = 10 m. rad		
			SEQUENCE #1	SEQUENCE #2	SEQUENCE #3	SEQUENCE #1	SEQUENCE #2	SEQUENCE #3
NO MANEUVER	RANGE ERROR meter	PEAK	8.25	8.48	9.21	16.38	16.85	18.32
		RMS	3.11	3.18	3.48	6.2	6.36	6.96
	ANGLE ERROR m.rad	PEAK	10.5	17.95	13.34	20.96	35.89	26.65
		RMS	3.91	5.26	4.28	7.82	10.51	8.56
30 m/sec² MANEUVER	RANGE ERROR meter	PEAK	8.17	9.13	9.07	16.31	17.50	18.19
		RMS	3.19	3.28	3.46	6.28	6.44	6.93
	ANGLE ERROR m.rad	PEAK	10.5	17.95	13.34	20.96	35.89	26.65
		RMS	3.90	5.26	4.31	7.82	10.51	8.59
60 m/sec² MANEUVER	RANGE ERROR meter	PEAK	8.14	9.85	8.72	16.27	18.21	17.74
		RMS	3.30	3.37	3.52	6.36	6.52	6.97
	ANGLE ERROR m.rad	PEAK	10.5	17.95	13.34	20.96	35.89	26.65
		RMS	3.89	5.25	4.35	7.80	10.50	8.63

TABLE 4.5   CRITICALLY DAMPED FILTER, g = 0.5

NOISE / MANEUVER			σ_R = 4 m, σ_θ = 5 m. rad			σ_R = 8 m, σ_θ = 10 m. rad		
			SEQUENCE #1	SEQUENCE #2	SEQUENCE #3	SEQUENCE #1	SEQUENCE #2	SEQUENCE #3
NO MANEUVER	RANGE ERROR meter	PEAK	7.79	8.58	8.15	15.31	17.43	16.39
		RMS	2.78	2.79	2.98	5.51	5.60	5.92
	ANGLE ERROR m.rad	PEAK	10.06	14.05	10.94	20.00	28.02	21.72
		RMS	3.44	4.37	3.70	6.89	8.73	7.44
30 m/sec² MANEUVER	RANGE ERROR meter	PEAK	8.24	10.01	8.15	15.76	17.70	15.91
		RMS	3.14	3.08	3.03	5.81	5.81	5.91
	ANGLE ERROR m.rad	PEAK	10.06	14.05	10.94	20.00	28.02	21.72
		RMS	3.43	4.43	3.79	6.88	8.78	7.52
60 m/sec² MANEUVER	RANGE ERROR meter	PEAK	9.15	12.10	8.52	15.50	19.78	15.13
		RMS	3.53	3.49	3.27	6.10	6.13	6.01
	ANGLE ERROR m.rad	PEAK	10.06	14.05	10.94	20.00	28.02	21.72
		RMS	3.43	4.49	3.93	6.84	8.82	7.64

TABLE 4.6   OPTIMAL FILTER, g = 0.7

NOISE MANEUVER			$\sigma_R = 4$ m, $\sigma_\theta = 5$ m. rad			$\sigma_R = 8$ m, $\sigma_\theta = 10$ m. rad		
			SEQUENCE #1	SEQUENCE #2	SEQUENCE #3	SEQUENCE #1	SEQUENCE #2	SEQUENCE #3
MANEUVER OFF / NO	RANGE ERROR meter	PEAK	12.99	14.28	12.01	25.95	28.58	24.05
		RMS	4.39	5.14	4.85	8.78	10.28	9.70
	ANGLE ERROR m.rad	PEAK	14.84	19.60	16.04	29.70	9.20	32.07
		RMS	6.01	7.11	6.34	12.01	14.23	12.63
MANEUVER 30 m/sec²	RANGE ERROR meter	PEAK	12.97	14.28	12.01	25.93	28.58	24.05
		RMS	4.41	5.15	4.85	8.80	10.29	9.70
	ANGLE ERROR m.rad	PEAK	14.80	19.60	16.04	29.66	39.20	32.07
		RMS	6.00	7.11	6.34	12.01	14.23	12.69
MANEUVER 60 m/sec²	RANGE ERROR meter	PEAK	12.96	14.28	12.01	25.92	28.58	24.05
		RMS	4.42	5.15	4.86	8.81	10.29	9.70
	ANGLE ERROR m.rad	PEAK	14.82	19.60	16.04	29.68	39.20	32.07
		RMS	6.00	7.11	6.35	12.00	14.23	12.69

TABLE 4.7   CRITICALLY DAMPED FILTER, g = 0.7

MANEUVER	NOISE		$\sigma_R = 4$ m, $\sigma_\theta = 5$ m. rad			$\sigma_R = 8$ m, $\sigma_\theta = 10$ m. rad		
			SEQUENCE #1	SEQUENCE #2	SEQUENCE #3	SEQUENCE #1	SEQUENCE #2	SEQUENCE #3
NO MANEUVER	RANGE ERROR meter	PEAK	10.84	11.14	10.29	21.60	22.32	20.65
		RMS	3.70	4.17	4.00	7.38	8.34	8.00
	ANGLE ERROR m.rad	PEAK	13.27	17.98	14.32	26.62	35.95	28.62
		RMS	4.98	5.89	5.20	9.96	11.77	10.41
30 m/sec² MANEUVER	RANGE ERROR meter	PEAK	10.82	11.14	10.29	21.59	22.32	20.65
		RMS	3.75	4.21	4.00	7.43	8.38	7.99
	ANGLE ERROR m.rad	PEAK	13.13	17.98	14.32	26.48	35.95	28.62
		RMS	4.97	5.89	5.22	9.95	11.78	10.43
60 m/sec² MANEUVER	RANGE ERROR meter	PEAK	10.78	11.25	10.29	21.54	22.32	20.65
		RMS	3.80	4.26	4.01	7.47	8.42	8.00
	ANGLE ERROR m.rad	PEAK	13.12	17.98	14.32	26.47	35.95	28.62
		RMS	4.97	5.89	5.24	9.95	11.78	10.45

TABLE 4.8   PERFORMANCE OF g-h-k FILTERS

NOISE AND COEFFICIENT SELECTION / MANEUVER			OPTIMAL						CRITICAL					
			g = 0.3		0.5		0.7		0.3		0.5		0.7	
			$\sigma_R=4$ $\sigma_\theta=5$	$\sigma_R=8$ $\sigma_\theta=10$	$\sigma_R=4$ $\sigma_\theta=5$	$\sigma_R=8$ $\sigma_\theta=10$	$\sigma_R=4$ $\sigma_\theta=5$	$\sigma_R=8$ $\sigma_\theta=10$	$\sigma_R=4$ $\sigma_\theta=5$	$\sigma_R=8$ $\sigma_\theta=10$	$\sigma_R=4$ $\sigma_\theta=5$	$\sigma_R=8$ $\sigma_\theta=10$	$\sigma_R=4$ $\sigma_\theta=5$	$\sigma_R=8$ $\sigma_\theta=10$
NO MANEUVER	RANGE ERROR meter	PEAK	6.12	12.43	9.21	18.32	12.01	24.05	8.29	14.95	8.15	16.39	10.29	20.65
		RMS	2.57	4.99	3.48	6.96	4.85	9.70	3.07	5.08	2.98	5.92	4.00	8.00
	ANGLE ERROR m.rad	PEAK	9.38	19.40	13.34	26.65	16.04	32.07	5.78	11.73	10.94	21.72	14.32	28.62
		RMS	2.94	5.99	4.28	8.56	6.34	12.68	2.41	4.86	3.70	7.44	5.20	10.41
30 m/sec² MANEUVER	RANGE ERROR meter	PEAK	8.44	11.79	9.07	18.19	12.01	24.05	11.97	16.06	8.15	15.92	10.29	20.65
		RMS	3.11	5.21	3.46	6.93	4.85	9.70	5.42	6.63	3.03	5.91	4.00	7.99
	ANGLE ERROR m.rad	PEAK	9.38	19.4	13.34	26.65	16.04	32.07	8.64	12.71	10.94	21.72	14.32	28.62
		RMS	3.24	6.23	4.31	8.59	6.34	12.69	3.09	5.34	3.79	7.52	5.22	10.43
60 m/sec² MANEUVER	RANGE ERROR meter	PEAK	13.04	16.39	8.72	17.74	12.01	24.05	19.09	22.02	8.52	15.13	10.29	20.65
		RMS	4.15	5.88	3.52	6.97	4.86	9.70	7.91	8.66	3.27	6.01	4.01	8.00
	ANGLE ERROR m.rad	PEAK	9.58	19.4	13.34	26.65	16.04	32.07	10.02	15.11	10.94	21.72	14.32	28.62
		RMS	3.74	6.63	4.35	8.63	6.35	12.69	4.39	6.38	3.93	7.64	5.24	10.45

g   :   0.3   0.5   and 0.7

h,k   :   optimal and critical damping methods

Trajectory   : $\begin{cases} \text{three trajectories shown in Figure 4.1} \\ \text{no maneuver, 30 m/sec}^2 \text{ maneuver and} \\ 60 \text{ m/sec}^2 \text{ maneuver} \end{cases}$

Noise Levels: $\begin{cases} \sigma_R = 4m \text{ and } \sigma_\theta = 5 \text{ m.rad} \\ \sigma_R = 8m \text{ and } \sigma_\theta = 10 \text{ m.rad} \end{cases}$

Examination of the results reveals the following observations:

(1)   Low values of g provide the best performance when the target is not maneuvering. But a target maneuver has a deteriorating effect.

(2)   Target maneuver has no effect on the errors at high values of g. However, they have very poor noise smoothing.

(3)   At low values of g, the optimal filter is better than the critically damped filter, whereas the opposite is true at high values of g. This is because the optimal filter becomes highly underdamped at high values of g.

(4)   For the particular noise and maneuver parameters chosen, a moderate value of g, *i.e.*, g = 0.5, seems to provide the best compromise.

From the results, it is apparent that a single value of g cannot provide a satisfactory performance for all types of trajectories. A small value of g is adequate for tracking nonmaneuvering targets but may lose track in case of a maneuver. On the other hand, a large value of g makes the performance insensitive to target maneuvers, but may lose track because of its poor noise smoothing capability.

4.5   Conclusions

The performance of fixed coefficient g-h-k filters for tracking maneuvering and nonmaneuvering targets was studied via simulation. Three different filter initialization schemes were evaluated in the light of good transient performance. The tracking capability of the filter was studied by varying the filter and target parameters. The main conclusion is that a single set of filter coefficients is not sufficient to provide an adequate performance for both maneuvering and nonmaneuvering targets. Thus, an adaptive filter suggests itself, whereby the coefficients are altered when changes in the trajectory are detected. A method of making the filter adaptive to target maneuvers is discussed in the next chapter.

CHAPTER 5
ADAPTIVE G-H-K FILTER FOR TRACKING MANEUVERING TARGETS

5.1   Introduction

The most commonly used tracking filters, $i.e.$, the Kalman filter, the g-h filter, the g-h-k filter, etc., operate satisfactorily provided the dynamic model on which the tracking algorithm is based is a correct representation of the true state of the target. Usually, the model is based on the assumption that the target follows a constant velocity or a constant acceleration trajectory. However, such a model cannot adequately describe a maneuvering target, not only because the constant acceleration assumption is invalid, but also due to the inherent uncertainty in the time at which the maneuver is initiated. Singer [3] has suggested that a maneuvering target is well modeled by a linear acceleration model driven by random noise of proper variance, and hence, a Kalman filter can be used. This filter maintains track throughout the maneuver, but suffers significant degradation when the target is not maneuvering. This is analagous to the conclusion drawn in the previous chapter that a single set of g-h-k filter coefficients cannot provide a satisfactory performance for both maneuvering and nonmaneuvering targets.

One approach towards resolving this problem is the use of adaptive trackers, wherein the filter senses deviations of the trajectory from the model and modifies the algorithm accordingly. An adaptive Kalman filter is proposed in [14] where the predicted estimate is the weighted sum of two Kalman predictors, each of which is based on a different model. The relative weights are based on a likelihood ratio which reflects the possible occurrence of a maneuver. In the adaptive estimator described in [15] and [16], target maneuvers are modeled as unknown random variables which are estimated along with the target state. The unsupervised tracking scheme

given in [17] is a feedback filter where the gains are selected to force orthogonality in the residual sequence. The gains themselves are computed using a stochastic approximation algorithm. A decision-directed adaptive tracker is explained in [18], where a maneuver is detected from the bias in the residual sequence of a Kalman filter. The filter is reinitialized and a different value is used for the system noise covariance when a maneuver is detected. Linear regression filters are used in [19] for tracking maneuvering aircraft targets in cartesian coordinates.

In this chapter, an attempt is made to design an adaptive g-h-k filter. The procedure is to use a fixed coefficient g-h-k filter with high noise smoothing capability when the target is not maneuvering. However, when a maneuver is detected, the coefficients are modified so as to provide a good transient performance. Target maneuvers are detected by processing the residual sequence of the g-h-k filter. The procedure is simulated and exercised on maneuvering and nonmaneuvering trajectories to ascertain its validity. All the programs used are listed in Appendix B.

5.2   Maneuver Detection

5.2.1   Principle of the Method

Using the same notation as in Chapter 3, a maneuvering target is modeled by the system equations

$$\underline{X}(n+1) = \Phi \, \underline{X}(n) + \underline{A}(n) \tag{5.1}$$

$$x_m(n) = H \, \underline{X}(n) + v(n) \tag{5.2}$$

where

$$\Phi = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad,$$

$$H = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

and the vector forcing function $\underline{A}(n)$ is included to account for the target maneuver. Assuming that the maneuver was initiated at $n = n_0$ where n_0 is unknown,

$$\underline{A}(n) = 0 \qquad \text{for } n < n_0 \, . \tag{5.3}$$

The filter equation is given by,

$$\hat{\underline{X}}(n+1/n) = \Phi \, \hat{\underline{X}}(n/n-1) + \Phi \, K \, \nu(n) \tag{5.4}$$

where $\nu(n)$ is the residual sequence,

$$\nu(n) = x_m(n) - H \, \hat{\underline{X}}(n/n-1) \, , \tag{5.5}$$

and the gain vector K is given by

$$K = \begin{bmatrix} g \\ h/T \\ 2k/T^2 \end{bmatrix} \, . \tag{5.6}$$

Consider a nonmaneuvering system given by,

$$\underline{Y}(n+1) = \Phi \underline{Y}(n) \tag{5.7}$$

$$y_m(n) = H \, \underline{Y}(n) + v(n) \tag{5.8}$$

with the same initial condition as that of (5.1). The corresponding prediction equation is,

$$\hat{\underline{Y}}(n+1/n) = \Phi \, \underline{Y}(n/n-1) + \Phi K \, \nu_1(n) \tag{5.9}$$

where $\nu_1(n) = y_m(n) - H \, \hat{\underline{Y}}(n/n-1)$. \hfill (5.10)

It can be easily seen that,

$$\hat{\underline{X}}(n/n-1) = \hat{\underline{Y}}(n/n-1) \tag{5.11}$$

$$\text{and} \qquad \nu(n) = \nu_1(n) \quad\Bigg\} \quad \text{for } n \leq n_0 \quad .$$

If the Kalman gains were used for the feedback gains in (5.8), it is well known that the residual sequence would be a zero mean white noise sequence. While this is not true in general for the g-h-k filter, under the condition that the filter is in steady state, it can be assumed that the sequence $\nu_1(n)$ is a zero-mean independent sequence. However, this property is dependent on the model (5.7) being a true representation of the trajectory. If the target

is maneuvering, the residual sequence develops a bias which reflects the maneuver. The residual sequence $\nu(n)$ can be written as

$$\nu(n) = \nu_1(n) + b(n) \tag{5.12}$$

where $b(n) = 0$ for $n \leq n_o$. $\tag{5.13}$

Therefore, detecting a bias reduces to the following hypothesis test:

$$
\begin{aligned}
H_o &: \text{ no maneuver } : \nu(n) = \nu_1(n) \\
H_1 &: \text{ maneuver } : \nu(n) = \nu_1(n) + b(n)
\end{aligned}
\tag{5.14}
$$

where $\nu_1(n)$ is an independent sequence. That is, detecting a maneuver is equivalent to detecting for the presence of a deterministic signal of unknown amplitude and time of arrival in a background of zero-mean white noise.

5.2.2 Equation for the Bias

Comparison of (5.2) and (5.8) gives

$$x_m(n) = y_m(n) - H \underline{Y}(n) + H \underline{X}(n) \ . \tag{5.15}$$

Combination of (5.5) and (5.15) yields ,

$$
\begin{aligned}
\nu(n) &= y_m(n) - H \underline{Y}(n) + H \underline{X}(n) - H \hat{\underline{X}}(n/n-1) \\
&= y_m(n) - H \hat{\underline{Y}}(n/n-1) + H(\underline{X}(n) - \underline{Y}(n)) - H(\hat{\underline{X}}(n/n-1) - \hat{\underline{Y}}(n/n-1)) \\
&= \nu_1(n) + H(\underline{X}_2(n) - \underline{X}_3(n)) \tag{5.16}
\end{aligned}
$$

where

$$\underline{X}_2(n) = \underline{X}(n) - \underline{Y}(n) \tag{5.17}$$

$$\underline{X}_3(n) = \hat{\underline{X}}(n/n-1) - \hat{\underline{Y}}(n/n-1) \ . \tag{5.18}$$

From (5.1) and (5.7), it is easy to show that,

$$\underline{X}_2(n+1) = \Phi \underline{X}_2(n) + \underline{A}(n) \tag{5.19}$$

and (5.4) and (5.9) can be manipulated to give,

$$\underline{X}_3(n+1) = \Phi(I - KH)\underline{X}_3(n) + \Phi K H \underline{X}_2(n). \tag{5.20}$$

Defining,

$$\underline{X}_4(n) = \underline{X}_2(n) - \underline{X}_3(n) \tag{5.21}$$

equations (5.19) and (5.20) can be combined to yield,

$$\underline{X}_4(n+1) = \Phi(I-K\,H)\underline{X}_4(n) + \underline{A}(n). \tag{5.21}$$

Comparison of (5.12), (5.16) and (5.21) shows that the bias sequence can be expressed as,

$$b(n) = H\,\underline{X}_4(n+1) . \tag{5.23}$$

Since the maneuvering and nonmaneuvering models assume the same initial conditions,

$$\underline{X}_4(0) = 0 . \tag{5.24}$$

Equations (5.22), (5.23) and (5.24) provide a model for the bias in the residual sequence.

The initial effect of a target maneuver can be adequately modeled by a sudden change in acceleration, *i.e.*,

$$\underline{A}(n) = \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix} \qquad \text{for } n = n_o \tag{5.25}$$

$$= \underline{0} \qquad \text{otherwise} .$$

Using (5.25), it can be shown that the z-transform of the bias is given by,

$$B(z) = \frac{a\,z^{-n_o}(z+1)}{z^3 - (3-g-h-k)z^2 + (3-2g-h+k)z - (1-g)} \tag{5.26}$$

Assuming that the filter coefficients are selected to provide critical damping according to (3.5) - (3.7), the bias sequence can be shown to be of the form,

$$b(n) = c\,T^2(n-n_o)^2\,\beta^{(n-n_o)} \tag{5.27}$$

where c is a constant. A large value of g results in a small value of β and hence a small bias. This is in agreement with the earlier result that a large value of g produces a fast reaction to changes in the trajectory.

5.2.3   Bias Detection

As stated before, the maneuver detection reduces to a hypothesis

test where it is required to detect the presence of a deterministic signal $b(n)$ and its time of occurrence n_o in a background of white noise. The classical decision theoretical method is to pass the residual sequence $\nu(n)$ through a matched filter [12] which is matched to,

$$m(n) = (nT)^2 \beta^n \qquad (5.28)$$

and subject the output to a threshold detector. Assuming that the detection must be completed within p steps of the occurrence of the maneuver, the impulse response of the matched filter is,

$$\gamma(n) = m(p - n) \qquad 0 \le n \le p \qquad (5.29)$$
$$= 0 \qquad \text{otherwise} .$$

It is desirable to implement the matched filter as a recursive filter so as to avoid excessive computation and storage. An impulse response of the type given in (5.29), requires a filter of order k. In order to simplify the implementation, the impulse response $\gamma(n)$ is approximated by an exponential, which can be synthesized as a first order recursive filter. Denoting the new impulse response as,

$$\lambda(n) = \alpha^n , \qquad (5.30)$$

the difference equation governing the matched filter is given by,

$$\mu(n+1) = \alpha \, \mu(n) + T \, \nu(n+1) \qquad (5.31)$$

where $\mu(n)$ is the output and the input is the residual sequence $\nu(n)$. The output sequence $\mu(n)$ is passed through a threshold detector. If the output exceeds the threshold at $n = \ell$, a maneuver is detected and the time of its occurrence is estimated as

$$\hat{n}_o = \ell - p . \qquad (5.32)$$

The remaining problems are the selection of proper values for p and α. A large p results in more maneuver data in the matched filter, thereby making it easier to detect, but the lag between the occurance of the maneuver and its detection may be unacceptable. If p is small, detection is based on

few samples containing maneuver information, and the performance could be quite poor. The value of α should be chosen so that $\lambda(n)$ approximates $\gamma(n)$ and is therefore a function of g.

5.3   Adaptive Filtering Scheme

The maneuver detection method explained in the previous section is utilized to design a g-h-k filter which is adaptive to target maneuvers. It is assumed that the target is initially on a straight line path and that it is being tracked with a g-h-k filter using a small value for g to provide good noise smoothing. The residual sequence $\nu(n)$ is passed through the first order filter given by (5.31) for maneuver detection. The output $\mu(n)$ of this filter is compared to a preselected threshold δ. If at $n = n_1$, $\mu(n) = \mu(n_1)$ exceeds the threshold δ, a target maneuver is indicated. In such a case, the filter coefficients are increased in order to provide a fast response to trajectory changes and the filter is reinitialized. In order to reinitialize the filter, the I_s latest measurements and smoothed values are stored. When a maneuver is detected, the filter is applied using the new coefficients and starting from the earliest available smoothed values. Since large values of coefficients also result in poor noise smoothing, the filter coefficients are switched back to the original values after I_t tracking intervals following the maneuver detection.

Subroutine GHKADA implements the above scheme for one radar coordinate. The input to the program transferred through arguments are the array of measurements XM, and the threshold δ for detecting the maneuver, denoted as THR. Other inputs transferred through a common statement are the number of samples N, the tracking interval DELT, the filter coefficients GN, HN, KN for the nonmaneuvering part, the coefficients GM, HM, KM for the maneuvering part, the detection filter weight α (see (5.31)) denoted as WEIGHT, and

and the values of I_s and I_t denoted as ISTORE and ITRANS, respectively. The outputs are the arrays XP, XV and XA containing predicted position, velocity and acceleration, respectively, and are transmitted as arguments.

Main program TSTADA is similar to TSTGHK developed in Chapter 4. It is used to find the maximum and the RMS values of the error in the predicted position for various trajectories generated using subroutine TRAJ.

5.4   Adaptive Filter Performance

To evaluate the performance of the adaptive scheme, the simulation program was exercised on typical trajectories described in Chapter 4. The experiment was repeated for various combinations of the filter coefficients and target maneuvers. A typical set of results is presented in Table 5.1. During the nonmaneuvering position of the trajectory the optimal coefficients with GN = 0.3 are used. The value of the detection filter weighting coefficient WEIGHT was chosen as 0.85, since this selection provides a good approximation to the matched filter when g = 0.3. The threshold for maneuver detection in range was set at δ = 4.0, which was selected based on preliminary experimental results. For the types of trajectory used, the target maneuver does not have an appreciable effect on the angle tracking filter. Therefore only the results of the range tracking filter are given. The table gives the peak range error with GN = 0.3 using the adaptive and the nonadaptive filter. The adaptive filter uses GM = 0.5 and 0.6 with both the optimal and critical damping coefficient selection schemes. The values of I_s and I_t used are 3 and 10, respectively.

The results of Table 5.1 and others not presented permit the following inferences.

(1)   The adaptive scheme reduces the maximum error for high acceleration maneuvers.

TABLE 5.1   PERFORMANCE OF ADAPTIVE FILTER

(PEAK RANGE ERROR IN METERS)

MANEUVER	NON ADAPTIVE FILTER	ADAPTIVE FILTER GM = 0.5		ADAPTIVE FILTER GM = 0.6	
		OPTIMAL	CRITICAL DAMPING	OPTIMAL	CRITICAL DAMPING
30 m/sec^2	12.92	13.02	12.92	15.96	13.49
60 m/sec^2	17.45	15.56	15.56	15.56	15.56
90 m/sec^2	22.09	18.58	18.58	18.58	18.58

(2) During the maneuver, the selection of the filter coefficients based on critical damping is better than than the optimal selection. This is to be expected since the optimal scheme is slightly underdamped and therefore, not ideal during transients.

(3) The selection of I_s is not critical. In fact, the simpler choice of $I_s = 1$ gives almost an identical performance to $I_s = 3$.

(4) The maximum error occurs before the maneuver is detected. Therefore, the adaptive scheme would work better if the maneuver were detected earlier.

CHAPTER 6
SUMMARY, CONCLUSIONS AND SUGGESTIONS FOR FUTURE EFFORTS

6.1   Summary

Chapter 2 is a review of the g-h-k tracking filter and presents all the relevant equations. The selection of the filter coefficients is discussed in Chapter 3. Two of the filter coefficients can be expressed in terms of the third based on performance criteria, which involve noise smoothing and speed of response. The purpose of Chapter 4 is twofold: (i) to select a filter initialization scheme for best transient performance and (ii) to select the filter coefficients for various amounts of noise and maneuver. Since the best choice of the filter coefficients for nonmaneuvering and for maneuvering targets is quite different, it seems logical to make the filter adaptive, that is, to change the coefficients as soon as a maneuver is detected. Chapter 5 proposes a technique for maneuver detection which is based on the bias in the residual sequence. The performance of the adaptive filter is evaluated through computer simulation.

6.2   Conclusions

The selection of the g-h-k filter coefficients is very important and must be a compromise between the conflicting requirements of good noise smoothing and of good transient capability. Assuming either critical damping or optimal transient response for a specified amount of smoothing, one can express h and k in terms of g, where g is chosen to satisfy the smoothing/transient compromise.

The performance of the fixed coefficient g-h-k filter is quite good for a nonmaneuvering target and is still adequate for a wide range of maneuvers. The filter initialization has a serious effect on the performance and a scheme is proposed based on a straight line fit through the

three measurements. At low values of g, the optimal filter is better than the critically damped filter, whereas the opposite is true at high values of g. This is because the optimal filter becomes very underdamped at high values of g. A large value of g provides the best performance when the target is maneuvering and should be used in conjunction with critical damping.

Trajectories were generated where a straight line path is followed by a high acceleration maneuver. It was shown that the maneuver introduces a bias in the residual sequence. A technique which approximates matched filtering followed by threshold detection was proposed to detect the maneuver. The detection requires at least eight samples after the maneuver starts. An adpative filtering scheme was suggested where the coefficients are selected for good smoothing performance prior to maneuvering and for good transient performance following a maneuver detection. It is necessary to reinitialize the filter when the coefficients are changed. The use of the adaptive filter reduces the tracking error during maneuvering. The maximum error occurs before detection of the maneuver.

6.3   Suggestions for Future Efforts

The performance of the adaptive filter is limited by the detection capability. It could be greatly improved if a maneuver were detected in fewer samples. Therefore, one may want to use a better approximation for the matched filter. The fixed threshold was determined experimentally, and a more systematic procedure is recommended.

All the study assumes a constant tracking interval. However, if the radar is used to track several targets, it may be advantageous to increase the sampling rate for a short interval following maneuver detection. One could consider other reinitialization techniques than the one described.

The filter was made adaptive to target maneuvers. In addition, one could also make the selection of the g-h-k coefficients a function of the RMS value of the noise to be estimated from the residual sequence.

APPENDIX A

PROGRA''S FOR EVALUATION OF FIXED COEFFICIENT G-H-K FILTERS

.nis appendix contains the FORTRAN listings of the programs which are used for the evaluation of g-h-k filters. The programs are: subroutines TRAJ, NOISE and GHKFIL, and main programs TSTINI and TSTGHK. All the programs are described in Chapter 4.

```
      SUBROUTINE TRAJ(R,A,N,DELT)
      DIMENSION R(1),A(1)
      READ 1,XO,YO,Z,VEL,THETA
      PRINT 2,XO,YO,Z,VEL,THETA
      READ 1,T1,T2,T3,AC
      PRINT 3,T1,T2,T3,AC
  1   FORMAT()
  2   FORMAT(5X,' XO YO Z VEL THETA ',5F12.5)
  3   FORMAT(5X,' T1 T2 T3 AC ',4F12.5)
      T=0.
      N=0
      CT=COS(THETA)
      ST=SIN(THETA)
      IF(AC.GT.1.) RAD=VEL*VEL/AC
 10   IF(T.GT.T1) GO TO 11
      N=N+1
      VT=VEL*T
      X=XO+VT*CT
      Y=YO+VT*ST
      R(N)=SQRT(X**2+Y**2+Z**2)
      A(N)=ATAN2(Y,X)
      T=T+DELT
      GO TO 10
 11   CONTINUE
      IF(AC.LT.1.) RETURN
      X1=X
      Y1=Y
 12   IF(T.GT.T2) GO TO 13
      ALPHA=VEL*(T-T1)/RAD
      N=N+1
      ANG=ALPHA-THETA
      X=X1+RAD*(ST+SIN(ANG))
      Y=Y1+RAD*(-CT+COS(ANG))
      R(N)=SQRT(X**2+Y**2+Z**2)
      A(N)=ATAN2(Y,X)
      T=T+DELT
      GO TO 12
 13   CONTINUE
      X2=X
      Y2=Y
      PHI=THETA-ALPHA
      CP=COS(PHI)
      SP=SIN(PHI)
 14   IF(T.GT.T3) RETURN
      N=N+1
      VT=VEL*(T-T2)
      X=X2+VT*CP
      Y=Y2+VT*SP
      R(N)=SQRT(X**2+Y**2+Z**2)
      A(N)=ATAN2(Y,X)
      T=T+DELT
      GO TO 14
      END
```

```
      SUBROUTINE NOISE(R,AZ,N,XMR,XMA,RSTD,ASTD)
      DIMENSION R(1),AZ(1),XMR(1),XMA(1),XN(400U)
      DATA SAVE/1476./
      XN(1)=SAVE
      N2=2*N
      CALL RANDN(XN,N2,0.0,1.0)
      DO 10 I=1,N
      XMR(I)=R(I)+RSTD*XN(I)
   10 XMA(I)=AZ(I)+ASTD*XN(I+N)
      SAVE=ABS(XN(N2))*43576
      RETURN
      END
```

```
      SUBROUTINE GHKFIL(XM,N,XP,VP,AP,DELT,G,H,K,RES,INIT)
      DIMENSION XM(1),XP(1),VP(1),AP(1),RES(1)
      REAL K
      GO TO(20,21,22),INIT
   20 XS=XM(3)
      VS=(XM(3)-XM(2))/DELT
      AS=0.
      GO TO 11
   21 XS=XM(3)
      VS=(XM(3)-XM(2))/DELT
      AS=(XM(3)-2.*XM(2)+XM(1))/(DELT**2)
      GO TO 11
   22 XS=(5.*XM(3)+2.*XM(2)-XM(1))/6.
      VS=(XM(3)-XM(1))/(2.*DELT)
      AS=0.
   11 CONTINUE
      DO 12 I=4,N
      XP(I)=XS+DELT*VS+AS*(DELT**2/2.)
      VP(I)=VS+DELT*AS
      AP(I)=AS
      RES(I)=XM(I)-XP(I)
      XS=XP(I)+G*RES(I)
      VS=VP(I)+RES(I)*H/DELT
   12 AS=AP(I)+RES(I)*(2.*K/DELT**2)
      RETURN
      END
```

```
MAIN PROGRAM TSTGHK

      DIMENSION R(200),AZ(200),XMR(200),XMA(200)
      DIMENSION RP(200),RV(200),RA(200),RES(200)
      DIMENSION AP(200),AV(200),AA(200),AES(200)
      REAL K
    1 FORMAT()
    2 FORMAT(5X,' G H K ',3F12.5)
    3 FORMAT(5X,' RSTD ASTD ', 2F12.5)
    4 FORMAT(5X,' INIT ',I5)
    5 FORMAT(/5X,' RESULTS FOR INOISE = ',I5//
     * 20X,' RANGE MEAN SQUARED ERROR   =   ',F12.5/
     * 20X,' RANGE R M S ERROR          =   ',F12.5/
     * 20X,' RANGE MAXIMUM ERROR        =   ',F12.5/
     * 20X,' ANGLE MEAN SQUARED ERROR   =   ',F12.5/
     * 20X,' ANGLE R M S ERROR          =   ',F12.5/
     * 20X,' ANGLE MAXIMUM ERROR        =   ',F12.5)
    7 FORMAT(5X,' DELT ',F12.5)
      READ 1,DELT
      PRINT 7,DELT
      READ 1,G,H,K
      PRINT 2,G,H,K
      READ 1,RSTD,ASTD
      PRINT 3,RSTD,ASTD
      READ 1,INIT
      PRINT 4,INIT
      CALL TRAJ(R,AZ,N,DELT)
      DO 10 INOISE=1,3
      SSQR=0.
      SSQA=0.
      ERMAXR=0.
      ERMAXA=0.
      CALL NOISE(R,AZ,N,XMR,XMA,RSTD,ASTD)
      CALL GHKFIL(XMR,N,RP,RV,RA,DELT,G,H,K,RES,INIT)
      CALL GHKFIL(XMA,N,AP,AV,AA,DELT,G,H,K,AES,INIT)
      DO 11 I=16,N
      RE=ABS(R(I)-RP(I))
      IF(RE.GT.ERMAXR) ERMAXR=RE
      SSQR=SSQR+RE**2
      AE=ABS(AZ(I)-AP(I))
      IF(AE.GT.ERMAXA) ERMAXA=AE
   11 SSQA=SSQA+AE**2
      XMSQR=SSQR/FLOAT(N-15)
      XMSQA=SSQA/FLOAT(N-15)
      RMSQR=SQRT(XMSQR)
      RMSQA=SQRT(XMSQA)
      PRINT 5,INOISE,XMSQR,RMSQR,ERMAXR,XMSQA,RMSQA,ERMAXA
   10 CONTINUE
      STOP
      END
```

MAIN PROGRAM TSTINI

```
      DIMENSION R(200),AZ(200),XMR(200),XMA(200)
      DIMENSION RP(200),RV(200),RA(200),RES(200)
      DIMENSION AP(200),AV(200),AA(200),AES(200)
      REAL K
   1  FORMAT()
   2  FORMAT(5X,' G H K ',3F12.5)
   3  FORMAT(5X,' RSTD ASTD ', 2F12.5)
   4  FORMAT(5X,' INIT ',I5)
   5  FORMAT(/5X,' RESULTS FOR INOISE = ',I5//
     * 20X,' RANGE MEAN SQUARED ERROR   =    ',F12.5/
     * 20X,' RANGE R M S ERROR          =    ',F12.5/
     * 20X,' RANGE MAXIMUM ERROR        =    ',F12.5/
     * 20X,' ANGLE MEAN SQUARED ERROR   =    ',F12.5/
     * 20X,' ANGLE R M S ERROR          =    ',F12.5/
     * 20X,' ANGLE MAXIMUM ERROR        =    ',F12.5)
   7  FORMAT(5X,' DELT ',F12.5)
      READ 1,DELT
      PRINT 7,DELT
      READ 1,G,H,K
      PRINT 2,G,H,K
      READ 1,RSTD,ASTD
      PRINT 3,RSTD,ASTD
      READ 1,INIT
      PRINT 4,INIT
      CALL TRAJ(R,AZ,N,DELT)
      DO 10 INOISE=1,3
      SSQR=0.
      SSQA=0.
      ERMAXR=0.
      ERMAXA=0.
      CALL NOISE(R,AZ,N,XMR,XMA,RSTD,ASTD)
      CALL GHKFIL(XMR,N,RP,RV,RA,DELT,G,H,K,RES,INIT)
      CALL GHKFIL(XMA,N,AP,AV,AA,DELT,G,H,K,AES,INIT)
      DO 11 I =4,15
      RE=ABS(R(I)-RP(I))
      IF(RE.GT.ERMAXR) ERMAXR=RE
      SSQR=SSQR+RE**2
      AE=ABS(AZ(I)-AP(I))
      IF(AE.GT.ERMAXA) ERMAXA=AE
   11 SSQA=SSQA+AE**2
      XMSQR=SSQR/FLOAT(N-15)
      XMSQA=SSQA/FLOAT(N-15)
      RMSQR=SQRT(XMSQR)
      RMSQA=SQRT(XMSQA)
      PRINT 5,INOISE,XMSQR,RMSQR,ERMAXR,XMSQA,RMSQA,ERMAXA
   10 CONTINUE
      STOP
      END
```

APPENDIX B

PROGRAMS FOR EVALUATION OF ADAPTIVE FILTER

This appendix contains the FORTRAN listings of the programs which are used for the evaluation of the adaptive g-h-k filter presented in Chapter 5. These programs are subroutine GHKADA and main program TSTADA. Other subroutines required for this study are TRAJ and NOISE which are listed in Appendix B.

```
      SUBROUTINE GHKADA(XM,XP,XV,XA,THR)
      COMMON/FLTPAR/N,DELT,GN,HN,KN,WEIGHT,ISTORE,GM,HM,KM,ITRANS
      DIMENSION XM(1),XP(1),XV(1),XA(1)
      DIMENSION XST(3,10),XS(3)
      DATA XST/30*0./
      REAL K,KN,KM
    2 FORMAT(5X,' MANEUVER DETECTED AT IS = ',I5/)
C  INITIALIZATION
      XS(1)=(5.*XM (3)+2.*XM (2)-XM (1))/6.
      XS(2)=(XM (3)-XM (1))/(2.*DELT)
      XS(3)=0.
      IS=3
      XDET=0.
      IMAN=0
      G=GN
      H=HN
      K=KN
      DO 10 I=1,3
   10 XST(I,1)=XS(I)
C PREDICTION ALGORITHM
   15 IS=IS+1
      IF(IS.GT.N) RETURN
      XP(IS)=XS(1)+DELT*XS(2)+XS(3)*(DELT**2/2.)
      XV(IS)=XS(2)+DELT*XS(3)
      XA(IS)=XS(3)
      RES =XM (IS)-XP(IS)
      IF(IMAN.EQ.1.OR.IS.LT.15) GO TO 12
C  MANEUVER DETECTION
      XDET=WEIGHT*XDET+DELT*RES
      IF(ABS(XDET).GT.THR) GO TO 11
   12 XS(1)=XP(IS)+G*RES
      XS(2)=XV(IS)+H*RES /DELT
      XS(3)=XA(IS)+K*RES *2./(DELT**2)
C  STORE ISTORE SMOOTHED VALUES
      DO 13 I=1,3
      IF(ISTORE.EQ.1) GO TO 13
      DO 14 J=ISTORE,2,-1
   14 XST(I,J)=XST(I,J-1)
   13 XST(I,1)=XS(I)
      IF(IMAN.EQ.0) GO TO 15
C  CHECK IF TRANSIENT PERIOD IS OVER
      IF(IS-IDET.LE.ITRANS) GO TO 15
      G=GN
      H=HN
      K=KN
      IMAN=0
      XDET=0.
      GO TO 15
C  REINITIALIZATION
   11 IDET=IS
      PRINT 2,IS
      IMAN=1
```

```
      G=GM
      H=HM
      K=KM
      DO 16 I=1,3
 16   XS(I)=XST(I,ISTORE)
      DO 17 I=ISTORE,1,-1
      PPX=XS(1)+DELT*XS(2)+XS(3)*(DELT**2/2.)
      PVX=XS(2)+DELT*XS(3)
      PAX=XS(3)
      RESXX=XM (IDET-I+1)-PPX
      XS(1)=PPX+G*RESXX
      XS(2)=PVX+H*RESXX/DELT
 17   XS(3)=PAX+K*RESXX*2./(DELT**2)
      GO TO 15
      END




      MAIN PROGRAM TSTADA

      COMMON/FLTPAR/N,DELT,GN,HN,KN,WEIGHT,ISTORE,GM,HM,KM,ITRANS
      DIMENSION RP(200),RV(200),RA(200)
      DIMENSION AP(200),AV(200),AA(200)
      DIMENSION XMR(200),XMA(200)
      DIMENSION R(200),AZ(200)
      REAL KM,KN
 1    FORMAT()
 2    FORMAT(20X,' TRACKING INTERVAL      =          ',F12.5)
 3    FORMAT(20X,' COEFFICIENTS : NONMANEUVERING ',3(F12.5,3X))
 4    FORMAT(20X,' COEFFICIENTS : MANEUVERING    ',3(F12.5,3X))
 5    FORMAT(5X,' MAXIMUM AND RMS ERRORS ='/
     * 20X,' RANGE MEAN SQUARED ERROR    =   ',F12.5/
     * 20X,' RANGE R M S ERROR           =   ',F12.5/
     * 20X,' RANGE MAXIMUM ERROR         =   ',F12.5/
     * 20X,' ANGLE MEAN SQUARED ERROR    =   ',F12.5/
     * 20X,' ANGLE R M S ERROR           =   ',F12.5/
     * 20X,' ANGLE MAXIMUM ERROR         =   ',F12.5)
 6    FORMAT(20X,' EXPONENTIAL FILTER WEIGHT = ',F12.5/
     *       20X,' RANGE THRESHOLD         =        ',F12.5/
     *       20X,' ANGLE THRESHOLD         =        ',F12.5)
 7    FORMAT(20X,' SAMPLES FOR REINITIALIZATION  ',I5/
     *       20X,' SAMPLES IN TRANSIENT          ',I5)
 8    FORMAT(20X,' RANGE STD. DEVIATION    =        ',F12.5/
     *       20X,' ANGLE STD. DEVIATION    =        ',F12.5)
 9    FORMAT(/5X,' RESULTS FOR INOISE = ',I5/)
 21   FORMAT(/5X,' RANGE TRACKING COMPLETE'/)
 22   FORMAT(/5X,' ANGLE TRACKING COMPLETE'/)
      READ 1,DELT
      PRINT 2,DELT
      READ 1,GN,HN,KN
      PRINT 3,GN,HN,KN
```

```
      READ   1,GM,HM,KM
      PRINT 4,GM,HM,KM
      READ   1,WEIGHT,RTHR,ATHR
      PRINT 6,WEIGHT,RTHR,ATHR
      READ   1,ISTORE,ITRANS
      PRINT 7,ISTORE,ITRANS
      READ   1,RSTD,ASTD
      PRINT 8,RSTD,ASTD
      CALL TRAJ(R,AZ,N,DELT)
      DO 10 INOISE=1,3
      PRINT 9,INOISE
      SSQR=0.
      SSQA=0.
      ERMAXR=0.
      ERMAXA=U.
      CALL NOISE(R,AZ,N,XMR,XMA,RSTD,ASTD)
      CALL GHKAD2(XMR,RP,RV,RA,RTHR)
      PRINT 21
      CALL GHKAD2(XMA,AP,AV,AA,ATHR)
      PRINT 22
      DO 11 I=16,N
      RE=ABS(R(I)-RP(I))
      IF(RE.GT.ERMAXR) ERMAXR=RE
      SSQR=SSQR+RE**2
      AE=ABS(AZ(I)-AP(I))
      IF(AE.GT.ERMAXA) ERMAXA=AE
   11 SSQA=SSQA+AE**2
      XMSQR=SSQR/FLOAT(N-15)
      XMSQA=SSQA/FLOAT(N-15)
      RMSQR=SQRT(XMSQR)
      RMSQA=SQRT(XMSQA)
      PRINT 5,XMSQR,RMSQR,ERMAXR,XMSQA,RMSQA,ERMAXA
   10 CONTINUE
      STOP
      END
```

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This report is concerned with the investigation of the g-h-k filter for tracking maneuvering targets. The selection of the filter coefficients is based on the amounts of noise and maneuver, and other system considerations such as critical damping or best transient response for specified smoothing. Several filter initialization schemes were tested. For low acceleration maneuvers, a considerable amount of smoothing can be achieved without losing track. However, in order to track severely maneu-

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vering targets, one must select coefficients which give a faster transient response at the expense of smoothing capability. Therefore, it is logical to use an adaptive filter with a good smoothing capability when the target is not maneuvering and a fast response during a target maneuver. Clearly, the main problem is to detect the maneuver in a reasonable amount of time. This can be done using a simplified matched filter followed by a threshold detector. The proposed adaptive filter was evaluated through computer simulation using typical trajectories. The performance of the adaptive filter is limited by the number of samples required by the detection filter and could probably be improved using a more complex maneuver detection filter.